An Improved Method for the Determination of Thermal Diffusivities

Chauncey Starr*
Harvard University, Cambridge, Massachusetts
(Received November 25, 1936)

The dynamic method of determining thermal diffusivities devised by King has been improved. A sinusoidal temperature is impressed on one end of a wire specimen, and the thermal diffusivity is determined from the measured decrement of the temperature wave traveling along the specimen. Elimination or determination of heat losses is unnecessary. Greater precision is possible with this method than with previous methods. The thermal diffusivity of a pure specimen of nickel, measured at 25°C, was 0.15885 cm²/sec., with a probable error of 0.06 percent. The thermal conductivity of nickel corresponding to this determination is 0.618 watts/cm °C.

DYNAMIC methods for the determination of the thermal conductivity of metals actually measure the thermal diffusivity, i.e., the ratio of the thermal conductivity to the thermal capacity per unit volume. Specific heat and density data permit the calculation of the thermal conductivity. In general, these methods involve subjecting one end of a bar specimen to a periodic or sudden temperature variation and a subsequent study of the temperature changes along the specimen. Since a simple-harmonic or sine wave of temperature is the only kind which is propagated without change of form, this is the periodic variation customarily used. A knowledge of the temperature wave characteristics (velocity and decrement) is sufficient to determine the thermal diffusivity.

Theory and Method

The present method is a variation of that devised by King.[1] One end of a long wire specimen is in contact with a heat source whose temperature varies sinusoidally. The other end is at the same temperature as the surroundings. The differential equation for linear heat flow is usually taken as

$$kd^2\theta/dx^2 = d\theta/dt + h\theta,$$

in which θ is the temperature difference between the specimen and the surrounding medium, h is the coefficient of surface heat loss, and k is the

thermal diffusivity. The solution of this equation subject to the boundary conditions $x = 0$, $\theta = \theta_1 + \theta_2 \cos \omega t$, and $x = \infty$, $\theta = 0$, is

$$\theta = \theta_1 e^{-mx} + \theta_2 e^{-nx} \cos (\omega t - px),$$

in which $km^2 = h$, $2kn^2 = (h^2 + \omega^2)^{\frac{1}{2}} + h^2$, $2kp^2 = (h^2 + \omega^2)^{\frac{1}{2}} - h^2$, and $p = 2\pi/Tv$, where T is the period of the wave and v its velocity. In King's method the velocity of the temperature wave is determined by dividing the distance between two thermojunctions placed on the specimen by the time lag of the heat wave in traveling from the first to the second, as determined from galvanometer deflections. The unknown constant h can be eliminated from the calculation of k by measuring the velocity for two separate periods of the heat wave. In the present method the decrement of the temperature wave is used rather than the velocity. This is determined from the ratio of the amplitudes of the wave at each thermojunction. The thermal diffusivity can be calculated from measurement of this decrement for two periods. If q_1 is the ratio of the two amplitudes for any one period T_1, $a = T_1/T_2$, $b = \ln q_2/\ln q_1$, and L is the distance between the thermojunctions, then from measurements made at two periods

$$k = \frac{\pi L^2}{T_1 \ln q_1 \ln q_2} \left(\frac{a^2 - b^2}{b^2 - 1}\right)^{\frac{1}{2}}.$$

The amplitude method has several advantages over the velocity method of King. The measurement of the amplitude decrement involves only the ratio of total galvanometer deflections and calibration of the galvanometer. The centering

* Charles A. Coffin Fellow and Research Fellow in physics.
[1] R. W. King, Phys. Rev. 6, 437 (1915). See also Ellis, Morgan and Sager, Renss. Poly. Inst. Bul. (Eng. and Sci. Series) No. 21 (1928) and No. 27 (1930).

of the galvanometer oscillation about the scale zero is not very important. The measurement of the wave velocity requires accurate setting of the galvanometer oscillation about the zero and some constant time lag device and chronograph for recording the instant of passage through this zero. It is thus apparent that the amplitude ratio can be determined with greater precision than the wave velocity. In both methods it is an essential condition that the mean temperature of the thermojunctions remain constant. However, it is easy to correct the amplitude measurement for any small change in thermal equilibrium, whereas this is very difficult in the velocity measurement. In addition, the amplitude measurement requires only one galvanometer instead of two, and does not require any chronographic devices.

An alternative method of finding k results from the measurement of the static decrement and the decrement for one period. With the heat source held at a constant temperature, the ratio of the temperatures at the two thermojunctions suffices for the calculation of m. Using one period, n is determined. From n and m, k can be found.

It should be noted that the $h\theta$ term of the differential equation involves the assumption that the heat loss from the specimen surface is proportional to and in time phase with the temperature of the surface. Aside from second order effects due to variation of the thermal constants with temperature, this assumption is only valid when the specimen is in a vacuum. Under any other circumstance, the thermal capacity of the surrounding medium results in a storage of heat, so that the temperature change of the medium lags that of the specimen. This produces an out of phase component of the heat loss that is mathematically difficult to treat. The effect is negligible for a gas, but is appreciable for a liquid medium.

The measurement of the distance between thermojunctions is a source of error common to all methods for determining thermal conductivity. The finite size of the thermocouples, and the distortion of heat flow due to the difference in thermal conductivities of the thermocouple wires and the specimen, make it difficult to find the true distance between the thermojunctions. Since the ratio of the thermal to the electrical
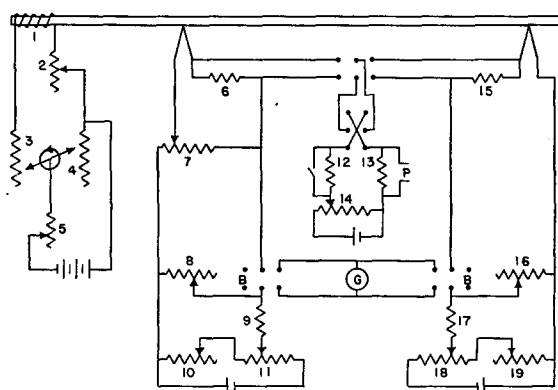

FIG. 1. Circuit diagram of apparatus.

conductivity is approximately the same for all metals, electric current and heat flow will be distorted similarly. This permits the determination of the effective distance between the thermocouples by electrical measurement. A current was sent through the specimen and the fall of potential between the two thermocouples was compared with that between two razor blades resting on the surface. Measurements were made for both directions of current flow. The distance between the razor blade marks was measured on a comparator. The effective distance between the two thermojunctions could thus be measured with a high precision.

APPARATUS

The circuit diagram of the apparatus used is shown in Fig. 1. The specimen is in the form of a wire usually about 2 to 3 mm in diameter and from 25 to 50 cm long. The periodic temperature variation at one end of the specimen is produced by means of a small heating coil through which passes an electric current varying according to the expression $I = I_0 |\sin (\omega t/2)|$. The heating coil consisted of 5 mil manganin wire wound tightly about the specimen and had a length of approximately 5 mm, and a resistance of slightly less than 10 ohms. The currents required to produce appropriate temperature changes were of the order of 0.2 amp. The heating coil was connected to a periodically varying resistance in series with a storage battery source. In order to maintain voltage stability, this resistance was designed to vary the current through the heating coil according to the above formula and at the

same time draw a constant current from the source. Two systems could be used, shunt or series. The shunt system, similar to that used by King, although easier to construct, has the disadvantage of drawing twice as much current as is required for the heating coil. Therefore the series system, as shown in the diagram, was used. Each variable resistance consisted of an Advance wire resistance bank connected to 72 taps arranged in a circle with a resistance arm rotating at constant angular velocity. The variable resistance (3) was designed according to the formula $r_3 = r_0(|\sin(\omega t/2)|^{-1} - 1)$, and the compensating resistance (4) according to the formula $r_4 = r_0 + r_0^2/r_3$. The value of r_0 was 10 ohms, and an auxiliary Advance resistance (2) was placed in series with the heating coil (1) to adjust it to this value. A resistance (5) in series with the storage battery adjusted the maximum value of the current. Although the heating coil current was varied in small increments, the thermal capacity of the heating coil and specimen was sufficient to integrate these and produce a smooth sine temperature form at the first thermojunction, even for the longest period used. An 1800 r.p.m. synchronous motor connected through variable reduction gearing operated the two rotating resistance arms. The periods used ranged from 1 to 6 minutes. Stopwatch measurements of the period of the rotating arms agreed so closely with the periods calculated from the synchronous speed of the motor and reduction gearing ratio, that the latter was used in the calculations.

The thermocouples were made of 5 mil Nichrome and Advance wire. These conducted a negligible amount of heat from the specimen. The e.m.f. produced at each thermojunction consists of the periodic sine component and an average constant component. A balancing circuit associated with each thermocouple neutralized the average component, so that the galvanometer followed the sine component only. Since the temperature variation at the first thermojunction is very much greater than that at the second, a shunt (7) was used to reduce the e.m.f. impressed on the galvanometer. After setting this shunt, both circuits were adjusted by means of the resistances (8) and (16) to the critical damping resistance for the galvanometer. The resistance

of the balancing circuits was so high as not to effect this adjustment. A double-pole-double-throw switch in each circuit served to connect either a resistance bridge (B) for the measurement of the circuit resistance or the galvanometer (G).

After every run each circuit was calibrated to determine the thermocouple e.m.f. per unit galvanometer deflection. In each circuit a precisely known 1 ohm manganin resistance (6) and (15) in series with the thermocouple was used to impress a known e.m.f. in the circuit. The calibration circuit consisted of a voltage divider (14), reversing switches, and a precision 5000 ohm manganin resistance (13). The voltage drop across this resistance was measured by a potentiometer (P). The impressed voltage in the thermocouple circuit could thus be measured to 0.01 percent. The series resistance (12) was in the circuit for the calibration of the second thermocouple, but was short-circuited for the calibration of the first.

The resistance values used were as follows: (7) 15 ohms, (8) 25 ohms, (16) 10 ohms, (9) (10) (14) (17) (18) 10,000 ohms, (11) (19) 100 ohms, (12) 20,000 ohms. The dry-cells used in the thermocouple and calibration circuits were left permanently connected. All the resistance elements, batteries, and switches of these three circuits were placed in one heat insulated box, to minimize temperature fluctuations. The apparatus was thus very free from any changes during a run.

The galvanometer used had a voltage sensitivity of 0.16μv/mm/m, and a critical damping resistance of 32 ohms. A 500 cm straight scale, one meter distant from the galvanometer mirror, and a single filament lamp, permitted sufficiently precise readings to be made, since the heating coil current and thermocouple shunt could be adjusted to spread the sine wave over the scale. The galvanometer calibration was linear within the experimental error and did not change appreciably with time. All readings were corrected for the noncurvature of the scale.

RESULT FOR NICKEL

Measurements were made on a nickel specimen about 3 mm in diameter and 27 cm long, mounted

vertically in a thick-walled standard copper pipe 3 inches in diameter. In order to keep the ambient temperature constant and uniform, the whole unit was placed in a constant temperature cabinet. Provision was made to evacuate the copper pipe if trouble was experienced from convection currents, but this was not found necessary. The mean distance between the two thermojunctions, measured electrically as described above, was 51.078±0.012 mm at 25°C.

Determinations were made with three different periods, and since any two were sufficient to calculate k, three values of k could be found, any two of which may be considered as independent determinations. During any one run, the mean temperature of the thermojunctions varied slightly due to uncontrolled temperature fluctuations (always less than 1 percent of the total deflection), but the following manner of averaging the readings corrected for this effect. Each total galvanometer deflection was found by adding the deflection on one side of the scale zero to the mean of the deflections on the other side, immediately preceding and succeeding. The deflections used in the calculations were the mean of sixteen deflections found as above, averaged in a manner similar to that described to compensate for any gradual amplitude change. This was found to be negligibly small. The maximum temperature difference between the specimen and its surroundings was about 5°C. Since the thermal diffusivity decreases about 0.1 percent per °C increase, the result given is an average about 25°C.

The value of k determined from the first and second periods was 0.15866, from the second and third periods, 0.15902, and from the first and third periods, 0.15887. Analysis of the equation for k shows that the greater the separation of the two periods, the smaller is the effect of errors in the measurement upon the computed value of k. For this reason the value determined from the first and third periods was given twice the weight of the other two. The probable error of the result also includes the probable error of the measurement of the distance between the thermojunctions. The weighted mean of the determinations of the thermal diffusivity of the nickel specimen at 25°C was

$$k = 0.15885 \pm 0.00009 \text{ cm}^2/\text{sec}.$$

The specimen was pure nickel, generously supplied by the Research Laboratory of the International Nickel Co., and contained carbon 0.009 percent, iron plus copper plus cobalt less than 0.01 percent, with a total nickel content greater than 99.98 percent. The specimen had been annealed in hydrogen at 870°C. The density was found to be 8.79 gr/cm³, and the electrical resistivity 7.21 microhm-cm at 22°C. Using the value for the specific heat of nickel given in the *International Critical Tables*, 0.4423 joules/gr °C at 25°C, the thermal conductivity calculated from the above value of the thermal diffusivity is $K = 0.618$ watt/cm °C.

The value of thermal diffusivity determined by these measurements is higher than that found by other methods. The measurement of Frazier[2] on a cold drawn nickel rod 99.23 percent pure, with a density of 8.85, and an electrical resistivity of 10.85 microhm-cm at 20°C, gave $k = 0.1500$. The lower purity and mechanical working of Frazier's specimen would cause a lower thermal conductivity, and thus might account for the lower k. The weighted mean value of the thermal conductivity of nickel according to the *International Critical Tables* is $K = 0.586$. The original values do not appear to have been properly weighted with regard to the state of purity and anneal of the nickel. The tables also give as the value for the electrical resistivity of very pure annealed nickel 7.23 microhm-cm at 20°C. This is in good agreement with the value for the specimen used. The most reliable value for the thermal conductivity of pure nickel is that of Schofield,[3] who found $K = 0.615$ for annealed nickel 99.2 percent pure, with a density of 8.79. This agrees well with the above computed value of K.

The writer is indebted to Professor P. W. Bridgman for his aid and encouragement in this work.

[2] R. H. Frazier, Phys. Rev. **39**, 515 (1932), and **40**, 592 (1932).
[3] F. H. Schofield, Proc. Roy. Soc. **A107**, 206 (1925).