The critical current of superconductors: an historical review

D. Dew-Hughes*

Oxford University, Department of Engineering Science, Parks Road, Oxford OX1 3PJ, UK

(Submitted May 4, 2001)
Fiz. Nizk. Temp. **27**, 967–979 (September–October 2001)

The most important practical characteristic of a superconductor is its critical current density. This article traces the history of the experimental discoveries and of the development of the theoretical ideas that have lead to the understanding of those factors that control critical current densities. These include Silsbee's hypothesis, the Meissner effect, the London, Ginsburg–Landau, and Abrikosov theories, flux pinning and the critical state, and the control of texture in high-temperature superconductors.
[DOI: 10.1063/1.1401180]

INTRODUCTION

The most important characteristic of any superconductor, from the viewpoint of practical applications, is the maximum electrical transport current density that the superconductor is able to maintain without resistance. This statement is equally true for large-scale applications, such as power transmission lines, electromagnets, transformers, fault-current limiters and rotating machines, as well as for small-scale electronic applications such as passive microwave devices and devices based on the Josephson effect. High lossless current densities mean that machines and devices can be made much smaller and more efficient than if made with a conventional resistive conductor. This was realized immediately upon the discovery of superconductivity; Onnes himself speculating on the possibility of magnet coils capable of generating fields of 10^5 G.[1] These early hopes were dashed by the inability of the then-known superconductors to sustain substantial currents, and applied superconductivity did not become a commercial reality until alloy and compound superconductors based on the element niobium were developed around 1960.[2] In the two following decades, intensive effort, primarily by metallurgists, led to the understanding of the factors that control critical currents and to the development of techniques for the fabrication of complex multifilamentary flexible conductors at economic prices. The discovery of the mixed copper oxide high-temperature superconductors initially produced a disappointment similar to that experienced by the pioneers of superconductivity. The superconducting characteristics of these materials introduced a new set of obstacles to achieving current densities of magnitudes sufficient for practical device applications. The difficulties involved in producing long lengths of high-current conductor from these materials are only just being overcome.

This article is not intended to be a review of everything that is known about critical currents in superconductors. Its aim is to trace the historical development of the understanding of the factors that control critical current density in superconductors. The significant experimental facts and theoretical ideas that have contributed to the present level of knowledge will be outlined, and the crucial contribution to the topic made by Lev Vasilievich Shubnikov will be highlighted.

THE EARLY YEARS 1911–1936

Within two years of his discovery of superconductivity in mercury, Onnes recorded that there was a ''threshold value'' of the current density in mercury, above which the resistanceless state disappeared.[3] This critical value was temperature dependent, increasing as the temperature was reduced below the critical temperature, according to the expression[4]

$$J_c(T)=J_c(0)(T_c-T)/T_c. \tag{1}$$

A similar behavior was observed in small coils fabricated from wires of tin and lead.[5] These represent the first-ever superconducting solenoids. Also noticed was the fact that the critical current density in the coils was less than that observed in short, straight samples of wire. This is the first instance of the phenomenon that was to plague the designers of superconducting magnets.

The following year Onnes reported on the influence of a magnetic field on the superconducting transition in lead: ''The introduction of the magnetic field has the same effect as heating the conductor.''[6] The existence of a critical magnetic field, above which superconductivity ceased to exist, was demonstrated. Surprisingly, perhaps because of the intervention of the First World War, Onnes failed entirely to make the connection between the critical current and the critical magnetic field. This connection was left to be made by Silsbee, as a consequence of his examining all of Onnes' published reports in great detail. Silsbee's hypothesis states, ''The threshold value of the current is that at which the magnetic field due to the current itself is equal to the critical magnetic field.''[7] From outside a conductor of circular cross-section, carrying a current I, the current appears to flow in a dimensionless line down the middle of the conductor. At a distance r away from a line current, there is a tangential magnetic field of strength

$$H(r)=I/2\pi r. \tag{2}$$

If the radius of the conductor is a, then the field at the surface of the conductor will be

$$H(a)=I/2\pi a \tag{3}$$

and the critical current, according to Silsbee's hypothesis, will be

$$I_c = 2\pi a H_c. \tag{4}$$

It should be noted that the critical current is thus not an intrinsic property of a superconductor, but is dependent upon the size of the conductor, increasing as the diameter of the conductor is increased. Conversely, the critical current density, also size dependent, decreases as the diameter of the conductor is increased:

$$J_c = 2H_c/a. \tag{5}$$

The experimental confirmation of Silsbee's hypothesis had to wait until after the end of the war. Both Silsbee[8] and the Leiden laboratory[9] carried out experiments on wires of differing diameters that did indeed confirm the correctness of the hypothesis. Tuyn and Onnes stated, "On the faith of these results obtained up till now we think we may accept the hypothesis of Silsbee as being correct." Silsbee's summary was, "It may therefore be concluded that the results of these experiments can be completely accounted for by the assumption of a critical magnetic field, without making use of the concept of critical currents."

Equation (2) is valid whatever the actual distribution of the current inside the conductor, and therefore Eq. (4) also holds for a hollow conductor of the same external radius. An ingenious extension of the Leiden experiments was to measure the critical current of a hollow conductor in the form of a film of tin deposited on a glass tube. An independent current was passed along a metal wire threaded through the tube. Depending on the direction of this current the critical current of the tin was either augmented or decreased, as the field at its surface resulted from both currents in the tin film and in the wire. This reinforced the validity of Silsbee's hypothesis.

At the same time, the Leiden laboratory was also making a study of the temperature dependence of the critical field in tin, with the result:[9,10]

$$H_c(T) = H_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]. \tag{6}$$

Also hysteresis in the superconducting transition was observed for the first time.[11] Hysteresis was subsequently observed in indium, lead, and thallium, and it was suggested that it might be an effect of purity, strain, or crystalline inhomogeneity.[12] It was decided that measurements on single crystals would be desirable, and in 1926 Shubnikov, who at that time was an expert in the growth of single crystals, joined the Leiden laboratory on a four-year secondment.

Meanwhile, in 1925, a new liquid helium laboratory was established at the Physikalische Technische Reichsanstalt in Charlottenberg. Chosen as the head of this laboratory was a former student of Planck, Walther Meissner. Meissner immediately instituted a program of work on superconductivity, but in order to avoid conflict with the Leiden group, this program concentrated on the superconducting transition metals, in particular tantalum and niobium.

At Leiden attention had now turned to binary alloys, one constituent of which was a superconductor and the other a nonsuperconductor. Not only did alloying often raise the transition temperature to well above that of the superconducting element, but these alloys also exhibited very high critical fields. These investigations culminated in the discovery that the Pb–Bi eutectic had a critical field of about 20 kG at 4.2 K, and its use to generate high magnetic fields was proposed.[13] This was actually attempted at the Clarendon laboratory in Oxford, to which Lindemann had recruited Simon, Kurti, and Mendelssohn as refugees from Nazi Germany. The attempt failed, as did a similar one by Keesom in the Netherlands. Resistance was restored at levels of magnetic field more appropriate to pure elemental superconductors. The conclusion was that the Silsbee's hypothesis was not valid for alloys.[14]

The studies on tin single crystals at Leiden had produced the puzzling results that, in a transverse field, resistance was restored at a value of field one-half of the critical field when the field was applied parallel to the axis of the crystal.[15] Von Laue, better known for his x-ray work, realized the significance of this result and suggested that it would be profitable to explore the distribution of magnetic field in the neighborhood of a superconductor.[16]

Meissner had already interested himself in this problem; he and others had considered the possibility of a supercurrent being essentially a surface current. In 1933 Meissner and Ochsenfeld published the results of their experiments in which they measured the magnetic field between two parallel superconducting cylinders. The enhancement of the field as the temperature was lowered below the critical temperature of the cylinders indicated that flux was being expelled from the body of the superconductors.[17]

Shubnikov had left Leiden in 1930 to take up a position at the Ukrainian Physicotechnical Institute in Kharkov, where he shortly became the scientific director of the newly established cryogenic laboratory. Liquid helium became available in the laboratory in 1933, and in the following year Rjabinin and Shubnikov gave confirmation of the Meissner effect in a rod of polycrystalline lead.[18]

The importance of this discovery of the Meissner effect to the understanding of superconductivity cannot be overemphasised. A perfect conductor will exclude flux if placed in an increasing magnetic field, but should retain flux if cooled to below its transition temperature in a magnetic field. The Meissner effect is the expulsion of flux from the body of a superconductor when in the superconducting state. The transition from the normal state to the superconducting state is path independent, and the superconducting state is thermodynamically stable. Armed with this knowledge it was possible to develop phenomenological theories of superconductivity. Being the more stable state below the transition temperature, the superconducting state has a lower energy than the normal state. It is possible to show, from simple thermodynamics, that the energy per unit volume of the superconducting state relative to the normal state is

$$\Delta G_{ns} = -\frac{1}{2}\mu_0 H_c^2. \tag{7}$$

This is in fact just the energy required to exclude the magnetic field from the superconductor.

Two phenomenological theories followed almost immediately from the discovery of the Meissner effect. The "twofluids'' model of Gorter and Casimir[19] was able to describe the influence of temperature on the properties of the superconducting state, and it is similar to the theory for liquid helium below its lambda point. In particular, the temperature dependence of the critical magnetic field, Eq. (6), can be derived from the two-fluid model. The London theory deals with the effect of magnetic fields upon the superconducting properties, and describes the spatial distribution of fields and currents within a superconductor.[20] The Londons showed that flux was not totally excluded from the body of a superconductor, but that it penetrated exponentially, from the surface, decaying over a characteristic length λ, the penetration depth

$$H(r)=H(0)\exp(-r/\lambda). \tag{8}$$

Associated with the gradient in field is a current

$$J(r)=\frac{\partial H}{\partial r}=-\frac{H(0)}{\lambda}\exp(-r/\lambda). \tag{9}$$

Note that this current has a maximum value at the surface, $r=0$, equal to $H_c(0)/\lambda$. This is the maximum current density that a superconductor can tolerate, and for lead, for example, with a critical field at 4.2 K of $\sim 4.2\times 10^4$ A/m and a penetration depth of $\sim$35 nm, this maximum current density is $\sim 1.2\times 10^{12}$ A/m^2. Another important result of the London theory was the conclusion that magnetic flux trapped by holes in a multiply connected superconductor, or within the body of the superconductor, must be quantized. The quantum of magnetic flux was shown to be $\Phi_0=h/q$, where h is Planck's constant and q is the charge of the carrier associated with superconductivity.

The groups at Oxford, Leiden, and Kharkov continued their studies on alloys. The addition of 4% Bi to Pb was sufficient to completely trap magnetic flux when an external field was reduced from above the critical field to zero.[21] In alloys of Pb–Ti and Bi–Ti, in increasing applied fields, flux began to penetrate at fields well below those at which resistance was restored.[22] Rjabinin and Shubnikov's work on single crystals of $PbTl_2$ clearly demonstrated the existence of two critical fields. Below the lower critical field, H_{k1} (in their notation), the alloy behaved as a pure metal superconductor, with no flux penetration. Above H_{k1} flux began to penetrate; penetration was completed at the upper critical field H_{k2}, at which point the resistance was restored. On reducing the field some hysteresis was observed, with a small amount of flux remaining in the sample at zero field.[23] Thus was type-II superconductivity recognized. It also appeared that Silsbee's hypothesis was obeyed by alloys, if the critical current was related to the lower critical field.

Mendelssohn essayed an ingenious explanation for the two critical fields, the hysteresis and flux trapping, with his ''sponge'' model.[24] This model postulated that a sponge or three-dimensional network of superconductor with a high critical field permeated the main body of the superconductor with a lower critical field. Flux penetration would commence once the external field exceeded the critical value for the body of the superconductor, but penetration would not be complete until the critical field of the sponge was reached. On reducing the field, the meshes of the sponge would trap flux, accounting for hysteresis. The nature of the sponge was not specified, but it was assumed that the meshes were of a dimension small compared to the penetration depth. Gorter[25] produced an alternative proposal, that the alloy superconductors subdivided into extremely thin regions, rather like a stack of razor blades, parallel to the applied field. This suggestion is remarkable in the light of Goodman's lamellar theory for type-II superconductivity.[26] However, even more prescient was Gorter's notion of a minimum size for the superconducting regions, foretelling the later concept of the coherence length.

Because in an ideal superconductor the flux expulsion is not complete, some surface penetration occurring, the energy required to expel the flux is less than that given by Eq. (7), and the actual critical field is slightly higher than that predicted from complete expulsion. This effect is barely noticeable in bulk superconductors, but can become appreciable when at least one dimension of the superconductor is comparable to, or smaller than, the penetration depth. H. London[27] showed that the critical field for a slab of superconductor, of thickness d, in an external field parallel to the faces of the slab is given by

$$H_f=H_c\left\{1-\frac{\lambda}{d}\tanh\frac{d}{\lambda}\right\}^{-1/2}. \tag{10}$$

When d is small compared to λ, this reduces to

$$H_f=\sqrt{3}\,\frac{\lambda}{d}H_c. \tag{11}$$

Thus thin films can remain superconducting to higher fields, and carry higher currents, than can bulk superconductors. This suggestion was verified experimentally by Shalnikov in 1938.[28] London suggested that, if the surface energy between normal and superconducting regions was negative, the superconductor would split into alternate lamellae of normal and superconducting regions, as suggested by Gorter. Fine filaments, of diameter less than the coherence length, are expected, by similar arguments, to have a higher critical field than that of the bulk. The Mendelssohn sponge could well be a mesh of fine filaments, with superconducting properties slightly better than those of the matrix. The filaments are assumed to result from inhomogeneities in the two-phase Pb–Bi alloys under investigation.

The picture emerging by mid-1935 was that, provided they were pure and free from strain, elemental superconductors exhibited complete flux exclusion, a reversible transition at a well-defined critical field, and a final state independent of the magnetization history. Alloys, on the other hand, showed gradual flux penetration starting at a field below, and finishing at a field somewhat higher, than the critical field typical of a pure element. In decreasing fields the magnetization of alloys was hysteretic, and residual trapped flux was often retained when the applied field had returned to zero. The so-called hard elemental superconductors such as Ta and Nb showed behavior similar to that of alloys.

The research at Kharkov continued with careful magnetization measurements on single and polycrystalline pure metals, and on single alloy crystals of Pb–Bi, Pb–In, Pb–Tl, and Hg–Cd. Shubnikov's final contribution to the critical current story was systematic magnetization measurements on a series of PbTl single crystals of differing compositions.[29]

These showed that the change from ideal to alloy behavior occurred at a particular concentration of the alloying addition. For lesser concentrations the alloy behaved as a pure metal. As the concentration was increased above this particular value, the field at which flux began to penetrate decreased, and the field at which resistance was restored increased, with increasing concentration of alloying element. A clear picture of the change from what is now recognized as type-I superconductor to type-II superconductor was presented, although there was an absence of cross-referencing between the Ukrainian and Western European work. The theoretical explanation of the two types of superconductor was still missing, as was any understanding of what really determined the critical current density. No further progress had been made on these two problems when once again work on superconductivity was frustrated by global conflict.

THEORETICAL ADVANCES 1945–1960

With the cessation of hostilities, renewed interest was taken in superconductivity. Helium gas was now much more readily available, its production having been accelerated by the needs of the US Navy for balloons. The development of the Collins liquefier allowed many more physics laboratories to indulge in studies at liquid-helium temperatures. However, the most startling advances were made on the theoretical front.

In 1950 Ginsburg and Landau, at the Institute for Physical Problems in Moscow, published their phenomenological theory.[30] They ascribed to the superconductor an order parameter, Ψ, with some characteristics of a quantum-mechanical wave function. Ψ is a function of temperature and magnetic vector potential. The Gibbs function is expanded in even powers of Ψ about the transition temperature, as in Landau's theory of phase transitions, and terms to describe the magnetic energy and kinetic energy and momentum of the electrons are included in their expression for the Gibbs function of a superconductor in an external field. At an external surface their theory reproduces the results of the London theory. They introduced a new parameter, characteristic of a particular superconductor, $\kappa = \sqrt{2}\lambda^2 q \mu_0 H_c/\hbar$. The problem that they set out to solve, following the earlier speculations of H. London, was that of the surface energy between superconducting and normal regions in the same metal. Their results showed quite clearly that, if κ were to have a value greater than $1/\sqrt{2}$, then superconductivity could persist up to fields in excess of the critical field, given by $H = (\kappa/\sqrt{2})H_c$. Ignoring the pre-War work on alloys, they stated that for no superconductor was $\kappa > 0.1$, and therefore this result was of no interest!

Pippard, with wartime experience of microwave techniques, was now at Cambridge, engaged in measurements of microwave surface resistance in metals and superconductors. The anomalous skin effect in impure metals had been explained by nonlocal effects. The behavior of an electron was not influenced by the point value of the electric and magnetic fields but by the value averaged over a volume of dimensions equal to the electron mean free path l. By analogy with the explanation for the anomalous skin effect in metals, Pippard suggested that a similar nonlocality was appropriate to superconductors. In the London theory, the current density at a point r is determined by the value of the magnetic vector potential $\mathbf{A}(r)$. In Pippard's nonlocal modification of the London theory[31] the current density at r is determined by $\mathbf{A}$ averaged over a volume of dimensions ξ_0. An electron traveling from a normal to a superconducting region cannot change its wave function abruptly; the change must take place over some finite distance. This distance is called the "range of coherence," ξ_0. Pippard estimated that, for pure (or clean) metals, $\xi_0 \approx 1\ \mu$m. The Pippard theory introduces modifications to the penetration depth. For a clean superconductor, clean in this case meaning that the normal electron mean free path $l \gg \xi_0$, the penetration depth is given by

$$\lambda_\infty = [(\sqrt{3}/2\pi)\xi_0 \lambda_L^2]^{1/3}, \tag{12}$$

where λ_L is the value of the penetration depth in the London theory.

For alloy, or dirty superconductors, in which $l \ll \xi_0$, the theory gives a new, much greater, value for the penetration depth,

$$\lambda = \lambda_L (\xi_0/l)^{1/2}, \tag{13}$$

and also a much reduced value for the coherence length,

$$\xi_d = (\xi_0 l)^{1/2}. \tag{14}$$

The Ginsburg–Landau κ can be shown to be approximately equal to λ/ξ, and for a dirty superconductor with l very small, i.e., high electrical resistivity in the normal state, κ can be quite large, e.g., ~ 25 for niobium-based alloys and compounds, and >100 for mixed oxide high-temperature superconductors.

The next theoretical development was the formulation of the Bardeen–Cooper–Schrieffer (BCS) microscopic theory for superconductivity.[32] This theory, for which the authors received the 1972 Nobel Prize for Physics, is now the accepted theory for conventional superconductors. In superconductors below the transition temperature, electrons close to the Fermi surface condense into pairs (Cooper pairs). These pairs are the charge carriers in superconductivity, and their charge q is equal to twice the charge on a single electron. The value of the flux quantum $\Phi_0 = h/2e = 2.07 \times 10^{-15}$ Wb. The pairs form under an attractive interaction mediated by lattice phonons. An energy gap appears in the excitation spectrum for electrons at the Fermi level. Electron pairs, lattice phonons, and energy gaps in superconductivity had been postulated previously, but Bardeen, Cooper, and Schrieffer were the first to put all of these together in one theoretical framework. The energy gap is related to the critical temperature:

$$2\Delta \approx 3.5 k T_c. \tag{15}$$

This represents the energy required to break up the Cooper pairs. It is possible to derive from this another estimate of the maximum current density, the depairing current. The depairing current density is that at which the kinetic energy of the superconducting carriers exceeds the binding energy of the Cooper pairs. It is then energetically favorable for the constituent electrons in a pair to separate and cease to be superconducting. The change in energy during scattering is maximized when the momentum change is maximized. This occurs when a carrier is scattered from one point on the

Fermi surface to a diametrically opposite one, in total reversal of direction. The carrier velocity is given by the sum of the drift and Fermi velocities; $v_d + v_f$ becomes $v_d - v_f$. The resulting change in kinetic energy is

$$\delta E_n = \frac{1}{2} m (v_d - v_f)^2 - \frac{1}{2} m (v_d + v_f)^2 = -2 m v_d v_f. \quad (16)$$

The breaking of a pair followed by scattering causes a change in energy

$$\delta E_s = 2\Delta - 2 m v_d v_f. \quad (17)$$

For spontaneous depairing to occur, δE_s must be negative, i.e., the drift velocity must be greater than $\Delta / m v_f$. The depairing current density J_d, which is just the drift velocity times the carrier density n and the carrier charge q, must therefore be greater than $J_d = nq\Delta / m v_f$. When appropriate substitutions are made this expression for J_d can be shown to reduce to H_c/λ, the previously quoted expression for the absolute maximum current density. Values of the depairing current density lie in the range $10^{12} - 10^{13}$ A/m^2.

Abrikosov, working in the same institute as Landau, made the fourth theoretical breakthrough in 1957.[33] He produced a mathematical solution of the Ginsburg–Landau equations for the case when $\kappa > 1/\sqrt{2}$. His solution showed that in a rising externally applied magnetic field, flux is excluded until a lower critical field, H_{c1}, is exceeded. Above H_{c1} flux penetrates in the form of flux vortices, or flux lines, each carrying a quantum of flux Φ_0, directed parallel to the field. The structure of these flux vortices is a normal core, of radius ξ, containing the flux that is supported by supercurrents circulating over a radius λ. As the applied field is increased, more flux penetrates until the density of the flux lines is such that the normal cores begin to overlap. This occurs at the upper critical field, $H_{c2} = \sqrt{2}\kappa H_{c0} = \Phi_0 / 2\pi\mu_0\xi^2$. The regime between the lower and upper critical fields is known as the "mixed state." The mutual repulsion between the flux vortices, in the absence of any other forces acting upon them, results in the formation of a triangular flux line lattice (FLL). The parameter of this lattice is $a_0 = 1.07(\Phi_0 / B)^{1/2}$, where B is the local value of the magnetic induction in the superconductor. Despite being published in translation, Abrikosov's paper took some time to be fully appreciated in the West.

In 1960 Gor'kov derived the constants in the Ginsburg–Landau theory from the BCS theory.[34] This trilogy of Russian theoretical work is collectively referred to as the GLAG (Ginsburg–Landau–Abrikosov–Gor'kov) theory. Superconductors with values of $\kappa > 1/\sqrt{2}$, which exhibit the mixed state, are known as type-II superconductors. For any superconductor, as the normal state mean free path of the electrons, l, is reduced, ξ gets smaller, λ gets larger, and κ increases. Alloying, by reducing l, raises κ. This explains Shubnikov's observation that the change from type-I to type-II behavior, or the onset of the Shubnikov phase, occurs at a particular alloy concentration.[29]

APPLIED SUPERCONDUCTIVITY 1960–1986

The experimentalists had not been idle during this period. New superconductors, showing a steady increase in critical temperature, had been discovered: the brittle compounds NbN (15 K) in 1941, V_3Si(17 K) in 1951, Nb_3Sn(18 K) in 1954, and the ductile alloys Nb–Zr (~11 K) in 1953 and Nb–Ti (~10 K) in 1961. All of these were type-II superconductors, with upper critical inductions well in excess of previously known materials. B_{c2} was about 12 T for Nb–Ti and 25 T for Nb_3Sn. The pioneers in this work were the groups at Westinghouse and Bell Telephone Laboratories.[2] Whereas the critical temperature and critical inductions were intrinsic properties of the superconductor, the critical current density was found to be strongly dependent upon the metallurgical state of the material. In two-phase alloys J_c was influenced by the size and dispersion of the second phase particles.[35] In the niobium-based ductile BCC alloys, it was found[36] that cold deformation significantly enhanced J_c.

The problem of fabricating wire from the brittle intermetallic Nb_3Sn was solved by filling niobium tubes with a mixture of Nb and Sn powders in the appropriate proportions, drawing to a fine wire, and reacting to form the compound.[37] This material had a current density of 10^9 A/m^2 in an induction of 8.8 T. Similar wire was wound into a solenoid which generated an induction of 2.85 T.[38] If the reaction to form the compound took place at the surface of the Nb and Sn particles, the compound could have formed as a three-dimensional network, just as envisaged by Mendelssohn for his sponge. Could the two-phase microstructure in the lead alloys, or the dislocations introduced by deformation of the ductile transition metal alloys,[39] constitute the elements of Mendelssohn's sponge? Or was there an alternative scenario? If the flux vortices in the mixed state were able to interact in some way with the microstructure, this interaction could impede both the ingress of flux in a rising field and the egress of flux in a falling field. This would lead to the magnetic hysteresis observed in the materials. Flux gradients resulting from nonuniform distributions of vortices can be equated with currents.

A current flowing in a superconductor in the mixed state will exert a Lorentz force on the flux vortices, $\mathbf{F}_{L(V)} = \mathbf{J} \times \mathbf{B}$ per unit volume of superconductor, or $\mathbf{F}_{L(l)} = \mathbf{J} \times \mathbf{\Phi}_0$ per unit length of vortex, where $\mathbf{\Phi}_0$ is a vector of strength $|\mathbf{\Phi}_0|$ directed along the vortex. The force acts in a direction normal to both flux and current. Unless otherwise prevented, the vortices will move in the direction of this force, and in so doing induce an electric field $\mathbf{E} = \mathbf{v} \times \mathbf{B}$, where $\mathbf{v}$ is the velocity of the vortices. The superconductor now shows an induced resistance, the value of which approaches that of the normal state, ρ_n, as the magnetic induction rises to B_{c2}, the upper critical induction.[40] The critical current is that current at which a detectable voltage is produced across the superconductor, and is therefore that current which just causes the vortices to move. If there is no hindrance to the motion of the vortices, then above B_{c1} the critical current is zero and the magnetization is reversible. The moving vortices do experience a viscous drag, originating from dissipation in the normal cores. This force $\mathbf{F}_\mathbf{v} = \eta \mathbf{v}$, where the coefficient of viscosity $\eta = \mathbf{\Phi}_0 \cdot \mathbf{B} / \rho_n$ (Ref. 40). If the vortices interact with microstructural features in the body of the superconductor, such as impurities, crystal defects, and second-phase precipitates, they can be prevented from moving and become pinned. The pinning force $\mathbf{F}_p$ is a function of the microstructure and the local value of the induction. If the current density is such that the Lorentz force is less than the pinning force, no movement of vortices will occur, and no voltage will be detected in the superconductor. If the current is increased to a value at which the Lorentz force exceeds the pinning force, vortices will move and a voltage will be detected. The critical current density is that value of the current density at which the vortices will begin to move, i.e., when $\mathbf{F}_L=\mathbf{F}_p$; thus giving $J_c=F_p/B$.

The sponge hypothesis was tested by studying an artificial sponge fabricated by impregnating porous borosilicate glass with pure metal superconductors.[41] The pores were interconnected and had a diameter of 3–10 nm. At the International Conference on the Science of Superconductivity, held at Colgate University the following year, the majority opinion swung in favor of pinning of flux vortices as the origin of magnetic hysteresis and the determinant of critical currents.[42] Nevertheless, Bean's experiments on the artificial sponge were important in leading to the concept of the critical state. Bean analyzed his results of magnetization measurements on the assumption that each filament of the sponge carried either its critical current, or no current at all. As the external field is raised, currents are induced in the outer filaments, shielding the inner filaments from the field. The field is able to penetrate only when the outer filament current attains its critical value. Filaments progressively carry the critical current until the flux has penetrated to the center of the sample. Reducing the field to zero leaves current flowing in all the filaments, and flux is trapped in the sample. Applying a field in the opposite direction causes a progressive reversal of the critical current in the filaments. Bean assumed a critical current in the filaments independent of field. The model can be modified to include a field dependence of the critical current, leading to a more realistic hysteresis curve.

The notion that the current in a superconductor is either everywhere equal to the critical current or zero transfers readily to the concept of a pinned Abrikosov vortex lattice. In terms of magnetization, as the external field is raised, vortices move into the superconductor. Their motion is resisted by the pinning forces, and local equilibrium is established. At each point on the invading flux front the Lorentz force exactly balances the pinning force, and the local current density is equal to the local value of the critical current density. The superconductor is in the critical state,[43] a term borrowed from soil mechanics. A heap of soil or sand, or snow on an alpine hillside, will come to equilibrium with a slope of gradient determined by gravity and friction. The addition of more material to the pile will cause a slide until equilibrium is re-established. The slope is metastable, and any disturbance will result in an avalanche. A similar situation obtains in a superconductor in the critical state. Any force acting so as to try to move a flux vortex is just opposed by an equal and opposite pinning force. An imposed disturbance, resulting from either a change in the external magnetic field or in a transport current, leads to a redistribution of flux until the critical state is restored. Spectacular flux avalanches, or jumps, have been observed in superconductors.[44] The one difference in the superconductor is that, as the pinning force is a function of the local induction, the slope of the flux front is not constant. Several empirical relations have been used to describe the dependence of critical current on local magnetic induction. Surprisingly the simplest possible relation, which, as it turns out, fits the data for commercial Nb–Ti conductor, namely $J_c(B)=J_c(0)(1-b)$, where $b=B/B_{c2}$ is the reduced induction, has been ignored.

The problem of calculating critical currents from known details of the microstructure bears some relation to that of calculating the mechanical properties of a structural alloy, or the magnetization curve of a magnetic material. In the case of structural alloys, elastic inhomogeneities impede the movement of crystal dislocations. In the case of magnetic materials, inhomogeneities in the magnetic properties impede the motion of domain walls. In superconductors the presence of inhomogeneity in the superconducting properties will impede the motion of flux vortices, and superconductors with strong pinning have been referred to as hard superconductors. The relation between microstructure, the properties of the vortex lattice, and critical currents has been the subject of several reviews, the most notable of which is that of Campbell and Evetts.[45]

Three factors must be considered in calculating pinning forces: the nature of the microstructural features, or pinning centers, responsible for pinning; the size, dispersion, and topography of these pinning centers; and the rigidity of the flux-line lattice. The nature of the pinning center determines the physical basis for the pinning force. A ferromagnetic precipitate will react very strongly with a flux line.[46] In most cases the pins are either nonsuperconducting precipitates or voids,[47] or regions whose superconductivity is modified, such as dislocation tangles, grain and subgrain boundaries. By passing through these regions the flux vortices reduce their length, and hence their energy, in the superconductor. The size of pins is important, since if they have a dimension significantly less than the coherence length ξ, their effectiveness is reduced by the proximity effect.[48] If they have dimensions of the order of the penetration depth λ, then local magnetic equilibrium within the pin can be established, magnetization currents will circulate around the pin, and the vortices will interact with these currents.[49] The number of pin–vortex interactions is determined by the dispersion of the pins. The topography decides whether the vortices, once unpinned, must cut across the pins or are able to slide round them.

The lattice rigidity is important as, if the pinning centers are randomly distributed, a rigid lattice will not be pinned. In practice the lattice is not rigid, and three responses to the pinning or Lorentz forces imposed upon it can be recognized. These forces may be such as to cause local elastic distortion of the lattice; they may exceed the yield strength of the lattice, causing local plastic deformation; or they may exceed the shear strength of the lattice. Whichever of these possibilities actually occurs provides the answer to what is known as the summation problem. If the lattice undergoes elastic distortion, the situation involves collective pinning.[50] The vortices are weakly pinned and the supercurrent densities are too low to be of practical interest. This situation will not be considered further. If the pinning forces are such as to cause local plastic deformation of the vortex lattice, the vortices will position themselves so as to maximize the pinning interaction. Each vortex can be assumed to act individually, and the global pinning force is just the direct sum of the individual forces. If the pinning forces are greater than the shear strength of the vortex lattice, some vortices may remain pinned, while the main part of the lattice shears past them.[51] However, this can only happen if there are paths down which the vortices can move without traversing any pins.[52]

If the experimental critical Lorentz force $\mathbf{J}_c \times \mathbf{B}$, determined from transport current measurements, is plotted versus the reduced value of the applied magnetic induction, b, it is found that, for a given sample, results at different temperatures lie on one master curve.[53] The master curve takes the form

$$J_c B = \mathrm{const} \cdot B_{c2}^{p+q} b^p (1-b)^q , \tag{18}$$

where the temperature dependence is incorporated in the temperature dependence of the upper critical induction. This is known as a scaling law. The values of the exponents p and q are peculiar to the particular pinning mechanism. Scaling laws are fundamental to flux pinning.[54] As an example, pinning by nonsuperconducting precipitates will be considered. If an isolated vortex intersects a spherical particle of normal material of diameter D, a volume of vortex core $D\pi\xi^2$ is removed from the system. Associated with the vortex core is an energy per unit volume $B_c^2/2\mu_0$. Thus the energy of the system is lowered by an amount $D\pi\xi^2 B_c^2/2\mu_0$. The force to move the vortex from a position in which it passes through the center of the particle, to a position outside the particle, is this change in energy divided by an interaction distance, which in this case is clearly the diameter of the particle. Thus the force to depin an isolated vortex from a normal particle is $\pi\xi^2 B_c^2/2\mu_0$. The total pinning force per unit volume is the single pin force multiplied by the number of active pins per unit volume. In this case this latter quantity is approximately equal to the total length of vortices per unit volume, B/Φ_0, multiplied by the volume fraction of particles, V_f. There is an additional effect to be taken into account. In the flux-line lattice, of reduced induction b, the density of superelectrons, and hence the superconducting condensation energy, is reduced by a factor $(1-b)$.[45] The pinning force per unit volume is thus

$$J_c B = \pi\xi^2 \frac{B_c^2}{2\mu_0}(1-b)\frac{B}{\Phi_0} V_f . \tag{19}$$

With the use of the expression for $B_{c2} = \Phi_0/2\pi\xi^2$, this becomes

$$J_c B = \frac{B_c^2}{4\mu_0} b(1-b) V_f . \tag{20}$$

The above derivation assumes only one vortex is pinned at each particle, and therefore the particle size must be less than the intervortex spacing. Based on the above expression, it is possible to make an estimate of the maximum pinning force, and hence the maximum current density. In order to maximize the pinning force, all vortices must be pinned over their entire length. This would require a microstructure consisting of continuous rods of nonsuperconductor, with diameter $\sim\xi$, parallel to the applied field, and at a spacing equal to that of the vortex lattice. In this case V_f is effectively 1, and the interaction distance is ξ. Taking Nb_3Sn as an example, with $B_c = 1$ T and $\xi = 3.6 \times 10^{-9}$ m, and considering that $b(1-b)$ has a maximum value of 0.25 at $b = 0.5$, i.e., at $B = 12.5$ T, we find

$$J_c = \frac{b(1-b)}{4.4\pi \times 10^{-7} \cdot 3.6 \times 10^{-9} B} = \frac{5.5 \times 10^{13} \cdot 0.25}{12.5} = 10^{12}\ \mathrm{A/m^2}. \tag{21}$$

Thus the maximum possible critical current density due to pinning is about one-tenth of the depairing current density. In practice, of course, it is impossible to achieve this idealized microstructure; maximum critical current densities due to pinning are about one-hundredth of the above estimate.

The two conventional superconductors in commercial production, the ductile transition metal alloy Nb–Ti, and the intermetallic compound Nb_3Sn, will now be examined in the light of the ideas expressed in the previous paragraphs. In order to confer stability, these conductors are fabricated as many fine filaments of superconductor in a copper matrix.[55]

In the case of Nb–Ti, rods of the alloy are inserted in a copper matrix, and drawn down, often with repeated bundling, drawing, and annealing schedules, to produce a multifilamentary composite wire. Extensive transmission electron microscope studies on pure Nb and V, and alloys of Nb–Ta, Nb–Zr, Nb–Ti, and Mo–Re, after cold deformation and annealing, have shown conclusively that, in these ductile metals, pinning is due to an interaction between flux lines and tangles of dislocations or cell walls, and not individual dislocations.[56] In these tangles the normal electron mean free path will be less than its value in the dislocation-free regions, and the local value of κ will be increased. This led to the idea of ΔK pinning,[57,58] the theory for which was developed by Hampshire and Taylor.[59] The superconducting filaments in Nb–Ti have a heavily deformed microstructure, with grains, subgrains, and nonsuperconducting α-Ti particles elongated in the direction of drawing. The current flow is parallel to this elongated microstructure, and the Lorentz force acting on the flux vortices is such as to drive them across the subgrain and normal particle boundaries. Pinning occurs at these boundaries and is a mixture of normal-particle and ΔK pinning, with a pinning function in which the critical Lorentz force $J_c B$ is proportional to $b(1-b)$.[54] The critical current is associated with the unpinning of flux vortices from these boundaries. The derivation of the pinning function is along similar lines to that described above for normal particles. Theory and experiment are well matched.[52] The above expression seems to hold whenever the critical current is determined by flux pinning with a density of pins less than the density of flux lines. The b term arises because, as the density of flux lines increases, so does the total length of line pinned. The $(1-b)$ term represents the decrease in superconducting order parameter with increasing induction.

The other commercial conductor is based on the intermetallic A15-type compound Nb_3Sn. Multifilamentary conductor is fabricated by some variant of the bronze process. In the original version of this process, rods of niobium are inserted in a copper/tin bronze ingot as matrix, and drawn, again with rebundling, to form a composite of fine niobium filaments in the bronze matrix. Reaction between the tin content of the bronze and the niobium at an elevated temperature converts the latter into Nb_3Sn filaments. This procedure is necessary, as the intermetallic compound is brittle and nondeformable. The critical Lorentz force in these materials is found to obey a scaling law similar to that postulated by Kramer,[51] namely $b^{1/2}(1-b^2)$. The critical current density increases as the grain size decreases, as would be expected if the pinning occurred at the grain boundaries, and as it does in Nb–Ti. The $(1-b^2)$ term has been taken to be indicative of some flux shearing process, as the C_{66} modulus of the flux-line lattice varies as $(1-b^2)$ at high values of b. It is not immediately obvious as to why these two types of material should behave in such different fashion, as their superconducting parameters and scale of microstructure are not vastly different. However examination of the microstructure of Nb_3Sn reveals it to be very different from that of Nb–Ti. This is not at all unexpected, due to the very different ways in which two microstructure are generated. That of bronze-processed Nb_3Sn consists of columnar grains whose axes are normal to the axes of the filaments.[60] The Lorentz force will act parallel to some of these boundaries, driving the flux lines along them rather than across them. A path is thus provided down which flux can shear, and the author has put forward a mechanism of flux-lattice dislocation-assisted shear.[52] Values of the critical Lorentz force predicted on this model are close both to the Kramer law and to observation; in addition the model predicts an inverse dependence of J_c on grain size, as is observed experimentally but not predicted on the Kramer theory. An alternative approach treats flux pinned at grain boundaries as Josephson vortices.[61] Transverse unpinning, with vortices crossing grain boundaries as in Nb–Ti, leads to the $b(1-b)$ scaling law, while longitudinal unpinning, with vortices traveling along grain boundaries as proposed for Nb_3Sn, leads to the $b^{1/2}(1-b^2)$ scaling law.

HIGH-TEMPERATURE SUPERCONDUCTORS

The immediate expectation from the discovery of the high-temperature, mixed copper oxide superconductors was that these materials could be exploited at 77 K to build electromagnets that would compete with permanent magnets, offering inductions in excess of 2 T. At low temperatures, the high critical fields would allow of competition with low-temperature superconductors, and the 21 T maximum induction available from existing A15 conductor would be exceeded. These high hopes have met with disappointment; the critical current densities, especially in high magnetic fields, are much less than those in low-temperature superconductors.

Typically, the critical current density as a function of applied induction for a high-temperature superconductor shows three regimes: an initial region in which the critical current decreases rapidly as soon as the field is turned on; a region, which can be linear, falling slowly with increasing field, and a third region in which the critical current falls to zero. The middle region may appear to be perfectly horizontal, indicating no dependence of critical current on applied field. It may also extend to very high fields, especially in Bi-2212 at temperatures below 20 K. An extreme example is a sample of spray-pyrolized TI-1223, in which the critical current density at 4.2 K is constant with field up to inductions of 40 T.[62] As the temperature is increased, all regions of the curve move to lower values of field and critical current density. In particular, the cutoff field decreases and the (negative) slope of the middle region increases. The significant fundamental differences between low-temperature and high-temperature superconductors are that the latter are anisotropic and have rather small coherence lengths. Structurally the mixed oxide superconductors are tetragonal, or nearly-tetragonal, with lattice parameters a and b lying in the range 0.375–0.395 nm and the c-axis parameter 3–12 times greater. This structural anisotropy leads to anisotropy in the physical properties of the compounds. In single crystals, the critical current density in the *ab* plane is many times greater than that in the c direction, normal to the *ab* plane. The superconducting coherence length ξ is small in these compounds; that in the c direction is just a few tenths of a nanometer in length, of similar magnitude to the region of crystallographic disturbance in the boundary between two grains. The consequence of this small range of coherence is that grain boundaries in high-temperature superconductors act as weak links, i.e., the superconducting wave functions in adjacent grains are only weakly coupled to one another. The overall critical transport current density in a superconductor is determined by whichever is the lesser of the *intragrain* or the *intergrain* current densities. The *intragrain* current density is controlled by flux pinning, the *intergrain* current density is a measure of the ability of current to flow from one grain to an adjacent grain. This latter depends upon the strength of the superconducting link across the boundary, and in the case of anisotropic superconductors, upon the relative orientation between the two grains.[63] The initial rapid drop in J_c with field is due to many weak links between grains being progressively switched off as the field is increased.[64]

The current that is left is now being carried by the few strong links that exist between the grains, and the number of these is relatively insensitive to magnetic field. The strength of supercurrent depends upon the proportion of grain boundaries that are strong links. Many models have been proposed to account for the manner in which current is transferred from grain to grain in anisotropic mixed oxide superconductors.[65] The conclusions from these models, confirmed by experience, is that the proportion of strong links between grains, and hence the intergrain current, is maximized by grain alignment. The material is textured so that the c axis of the grains is close to being normal to the direction of current flow, and that the *ab* planes of the grains are in near parallelism to one another. In effect, the conductor must be as close to being a single crystal as possible.

Once a degree of texture has been established, the current density is further determined by flux pinning. A fully textured material will carry no appreciable current density if the pinning is weak. Conversely, a material with strong pinning will also have a low critical current density if there is no texture. In anisotropic materials the pinning of flux is also anisotropic.[66] The pinning strength is a function of the direction of an external magnetic field relative to the *ab* planes of the superconductor. The critical current density is much higher with the field parallel to the *ab* planes than when it is perpendicular to them. The high-temperature superconducting compounds consist of groups of one, two, or three copper oxide layers, which are responsible for the superconductivity, separated by layers of other oxides that are essentially insulating. With the field lying parallel to the *ab* planes, the vortices will tend to place themselves in the insulating layers. The pinning mechanism, known as intrinsic pinning, is similar to that by normal particles as discussed above for low-temperature superconductors. The maximum critical current density should be of the same magnitude as that estimated in Eq. (20). The density of pins is much greater than the density of flux lines, explaining the relative insensitivity of the current density to external magnetic field in the middle region of the J_c versus B curve. When the applied field is normal to the *ab* planes, the intrinsic pinning no longer acts to hinder flux-line motion; the critical current densities are much lower than when the field is parallel to the planes. The situation is made worse by the fact that flux lines normal to the *ab* planes tend to split into "pancakes."[67] This tendency is greater the greater the ratio of nonsuperconducting oxide layer thickness to superconducting oxide layer thickness, and hence the degree of anisotropy in the material. The anisotropy can be reduced, and flux pinning can be enhanced, by chemical substitution that distorts the crystal structure, by the addition of nonsuperconducting phases, and by irradiation.

As the applied field continues to increase, a value is reached at which the critical current falls to zero. This is the irreversibility field, above which it becomes impossible to pin flux. Irreversibility in magnetization experiments also disappears. The magnitude of the irreversibility field decreases as the anisotropy and tendency to form pancake vortices increases. There is controversy as to the origin of the irreversibility field. Arguments persist as to whether it is caused by flux-lattice melting or by thermally activated depinning. What is interesting is that the critical Lorentz force in high-temperature superconductors in many cases follows scaling laws similar to those found for low-temperature superconductors. The one difference is that the reduced induction used in the scaling laws is that relative to the irreversibility field rather than the upper critical field. There are many examples of this in the literature. Scaling with the irreversibility field indicates that this field is an intrinsic property of the flux-line lattice.

The recently discovered superconductor MgB_2, with a critical temperature of 39 K,[68] appears to be a conventional low-temperature superconductor, with well-coupled grains and strong bulk pinning.[69] Transport current densities of 10^8 A/m^2, measured in self-field at 4.2 K, have been reported in wires fabricated from this material.[70] The nature of the pinning sites has not yet been determined.

CONCLUSIONS

The history of the experimental facts, and the theories developed therefrom, that have defined the understanding of the factors that control critical currents in both low- and high-temperature superconductors, has been delineated. At several critical stages opportunities have been missed. Onnes failed to connect critical fields with critical currents. The Meissner effect was discovered rather later than it ought to have been. In the late 1930s there was a lack of cooperation between the Leiden and Oxford groups on the one hand, and the Kharkov group on the other hand. Ginsburg and Landau dismissed the possibility of superconductors having values of $\kappa > 1/\sqrt{2}$. Abrikosov's ideas were slow to be appreciated. One is tempted to ask, "Would the first proper applications of superconductivity, in high field magnets, have arisen earlier than ~1960 if these delays had not occurred?" The answer is almost certainly "no." The applications were conditional upon the discovery and development of materials with the ability to carry high currents in high magnetic fields. These discoveries did not rely upon any phenomenological or theoretical developments, but were, as are so many useful discoveries, purely empirical.

The critical current density in both low-temperature and high-temperature superconductors is controlled by their microstructure. Flux pinning in the ductile alloys based on niobium occurs at dislocation tangles, subgrain boundaries, and interfaces with nonsuperconducting second phases α-Ti. Flux shear along columnar grain boundaries seems to be the controlling mechanism in the bronze-route A15 materials. In the high-temperature superconductors microstructural control must provide both a high degree of texture and flux pinning. The next challenge will be to control the microstructure of MgB_2.

For the pre-War history of superconductivity, I have drawn heavily upon P. Dahl's book *Superconductivity*.[71]

*E-mail: david.dew-hughes@eng.ox.ac.uk

[1] H. K. Onnes, Supplements to the Communications of the Physical Laboratory of Leiden University **34b**, 55 (1913).
[2] J. K. Hulm, J. E. Kunzler, and B. T. Matthias, Phys. Today **34**(1), 34 (1981).
[3] H. K. Onnes, Communications of the Physical Laboratory of Leiden University **133a**, 3 (1913).
[4] H. K. Onnes, *ibid.* **133b**, 29 (1913).
[5] H. K. Onnes, *ibid.* **133d**, 51 (1913).
[6] H. K. Onnes, *ibid.* **139f**, 66 (1914).
[7] F. B. Silsbee, Journal of the Washington Academy of Sciences **6**, 79 (1916).
[8] F. B. Silsbee, Proc. Natl. Acad. Sci., USA **13**, 516 (1927).
[9] W. Tuyn and H. K. Onnes, Communications of the Physical Laboratory, University of Leiden, **174a**, 21 (1926).
[10] W. Tuyn and H. K. Onnes, J. Franklin Inst. **201**, 379 (1926).
[11] W. J. De Haas, G. J. Sizoo, and H. K. Onnes, Physica **5**, 447 (1925).
[12] G. J. Sizoo, W. J. De Haas, and H. K. Onnes, in *Proceedings of the Amsterdam Akademie der Wetenschappen* **29**, 221 (1926).
[13] W. J. De Haas and J. Voogd, *ibid.* **32**, 874 (1929); **33**, 262 (1930).
[14] W. H. Keesom, Physica **2**, 36 (1935).
[15] W. J. De Haas and J. Voogd, in *Proceedings of the Amsterdam Akademie der Wetenschappen* **34**, 63 (1931).
[16] M. Von Laue, Phys. Z. **33**, 793 (1932).
[17] W. Meissner and R. Ochsenfeld, Die Naturwissenschaften **21**, 787 (1933).
[18] J. N. Rjabinin and L. V. Shubnikov, Nature (London) **134**, 286 (1934).
[19] C. J. Gorter and H. B. J. Casimir, Phys. Z. **35**, 963 (1934).
[20] F. London and H. London, Proc. R. Soc. London, Ser. A **149**, 71 (1935).
[21] T. C. Keely, K. Mendelssohn, and J. R. Moore, Nature (London) **134**, 773 (1934).
[22] W. J. De Haas and J. M. Casimir-Jonker, *ibid.* **135**, 30 (1935).
[23] J. N. Rjabinin and L. V. Schubnikov, Phys. Z. Sowjetunion **7**, 122 (1935).
[24] K. Mendelssohn and J. R. Moore, Nature (London) **135**, 826 (1935).
[25] C. J. Gorter, Physica **2**, 449 (1935).
[26] B. B. Goodman, IBM J. Res. Dev. **6**, 63 (1962).
[27] H. London, Proc. R. Soc. London, Ser. A **152**, 650 (1935).
[28] A. Shalnikov, Nature (London) **142**, 74 (1938).
[29] L. V. Schubnikov *et al.*, Phys. Z. Sowjetunion **10**, 165 (1936).
[30] V. L. Ginzburg and L. D. Landau, Zh. Éksp. Teor. Fiz. **20**, 1064 (1950).
[31] A. B. Pippard, Proc. R. Soc. London, Ser. A **216**, 547 (1953).
[32] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **108**, 1175 (1957).

[33] A. A. Abrikosov, Zh. Éksp. Teor. Fiz. **5**, 1442 (1957) [Sov. Phys. JETP **5**, 1174 (1957)].
[34] L. P. Gor'kov, *ibid.* **10**, 998 (1960).
[35] J. D. Livingston, Phys. Rev. **129**, 1943 (1963); Acta Metall. **11**, 1371 (1963); J. Appl. Phys. **34**, 3028 (1963); Rev. Mod. Phys. **36**, 54 (1964).
[36] M. S. Walker, R. Stickler, and F. E. Werner, in *Metallurgy of Advanced Electronic Materials*, edited by G. E. Brock [Interscience, New York (1963), p. 49].
[37] J. E. Kunzler, E. Buehler, F. S. L. Hsu, and J. E. Wernick, Phys. Rev. Lett. **6**, 89 (1961).
[38] L. C. Salter *et al.*, in *High Magnetic Fields*, H. H. Kolm *et al.* (Eds.), John Wiley, New York (1962), p. 344.
[39] R. Schaw and D. E. Mapother, Phys. Rev. **118**, 1474 (1960).
[40] Y. B. Kim, C. F. Hempstead, and A. R. Strnad, Phys. Rev. **139**, A1163 (1965).
[41] C. P. Bean, M. V. Doyle, and A. G. Pincus, Phys. Rev. Lett. **9**, 93 (1962).
[42] See Rev. Mod. Phys. **36**, (1964).
[43] C. P. Bean, Phys. Rev. Lett. **8**, 250 (1962); Y. B. Kim, C. F. Hempstead, and A. R. Strnad, Phys. Rev. **129**, 528 (1963).
[44] J. E. Evetts, A. M. Campbell, and D. Dew-Hughes, Philos. Mag. **10**, 339 (1964).
[45] A. M. Campbell and J. E. Evetts, Adv. Phys. **21**, 199 (1972).
[46] T. H. Alden and J. D. Livingston, J. Appl. Phys. **37**, 3551 (1966).
[47] E. J. Kramer and H. L. Freyhardt, J. Appl. Phys. **51**, 4930 (1980).
[48] J. Friedel, P. G. de Gennes, and J. Matricon, Appl. Phys. Lett. **2**, 119 (1963).
[49] A. M. Campbell, J. E. Evetts, and D. Dew-Hughes, Philos. Mag. **18**, 313 (1968).
[50] A. I. Larkin and Yu. N. Ovchinnikov, J. Low Temp. Phys. **34**, 409 (1979).
[51] E. J. Kramer, J. Appl. Phys. **44**, 1360 (1973).
[52] D. Dew-Hughes, Philos. Mag. B **55**, 459 (1987).
[53] W. A. Fietz and W. W. Webb, Phys. Rev. **178**, 657 (1969).
[54] D. Dew-Hughes, Philos. Mag. **30**, 293 (1974).
[55] M. N. Wilson, C. R. Walters, J. D. Lewin, and P. F. Smith, J. Phys. D **3**, 1518 (1970).
[56] A. V. Narlikar and D. Dew-Hughes, Phys. Status Solidi **6**, 383 (1964); J. Mater. Sci. **1**, 317 (1966).
[57] D. Dew-Hughes, Mater. Sci. Eng. **1**, 2 (1966).
[58] D. Dew-Hughes and M. J. Witcomb, Philos. Mag. **26**, 73 (1972).
[59] R. G. Hampshire and M. T. Taylor, J. Phys. F: Met. Phys. **2**, 89 (1972).
[60] C. S. Pande, *Metallurgy of Superconducting Materials*, edited by T. S. Luhman and D. Dew-Hughes, Academic Press, New York (1979), p. 171.
[61] J. McDonald and E. Barzi, IEEE Trans. Appl. Supercond. **11**, 3884 (2001).
[62] D. T. Ryan *et al.*, IEEE Trans. Magn. **32**, 2803 (1996).
[63] D. Dimos *et al.*, Phys. Rev. Lett. **61**, 219 (1988).
[64] R. L. Peterson and J. W. Ekin, Phys. Rev. B **42**, 8014 (1990).
[65] L. N. Bulaevskii *et al.*, Phys. Rev. B **45**, 2545 (1992); B. Hensel *et al.*, Physica C **205**, 329 (1993).
[66] P. H. Kes *et al.*, Phys. Rev. Lett. **64**, 1063 (1990).
[67] J. R. Clem, Phys. Rev. B **43**, 7837 (1991).
[68] J. Nagamatsu *et al.*, Nature (London) **410**, 63 (2001).
[69] M. Kambara *et al.*, Semicond. Sci. Technol. **14**, L5 (2001).
[70] B. A. Glowacki *et al.*, *ibid.* **14**, 193 (2001).
[71] P. F. Dahl, *Superconductivity, its Historical Roots and Development from Mercury to the Ceramic Oxides*, American Institute of Physics, New York (1992).

This article was published in English in the original Russian journal. Reproduced here with stylistic changes by AIP.