fields. In the case of integer spin the right hand side of (7) must be slightly more complicated to take the auxiliary conditions into account.


J. Schwinger, Phys. Rev. 91, 713 (1953). Translated by D. ter Haar

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COMPARISON OF THE MACROSCOPIC THEORY OF SUPERCONDUCTIVITY WITH EXPERIMENTAL DATA

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GOR' KOV has recently shown that the macroscopic equations for superconductors, established earlier by Landau and the author (see also references 3 and 4) follow from the current microscopic theory of superconductivity. He obtained then an essentially new result, namely a confirmation that the charge \( e_{\text{eff}} \) which occurs in these equations is equal to twice the electronic charge, \( 2e \). This result has an obvious physical meaning since the charge of a Cooper pair is just equal to \( 2e \). Meanwhile, the charge \( e_{\text{eff}} \) was previously usually put equal to \( e \). It is in that connection advisable to consider a comparison of the macroscopic theory with experiments, putting \( e_{\text{eff}} = 2e \). The parameter \( \kappa \) entering into the theory is then equal to

\[
\delta = \frac{1}{2} \delta_\infty \sqrt{\frac{T_c}{\Delta T}} \quad H_{\text{CM}} = \frac{dH_{\text{CM}}}{dT} c \Delta T,
\]

\[
x = 1.08 \cdot 10^7 \left( \frac{dH_{\text{CM}}}{dT} c T_c^2 \right) - \frac{1}{2} \delta_\infty,
\]

we have near \( T_c \)

\[
\delta = \delta_\infty \left( 1 - \left( T/T_c \right)^{1/4} \right),
\]

For tin

\[
\left( T_c = 3.73^\circ, \quad \left| \frac{dH_{\text{CM}}}{dT} \right| = 151, \quad \delta_\infty = 5.1 \cdot 10^{-5} \text{cm} \right)
\]

we have thus \( \kappa = 0.158 \). The limiting field for supercooling \( H_{\text{CM}} \) is for such a value of \( \kappa \) equal to \( H_{\text{CM}} / H_{\text{CM}} \approx 0.224 \). Experimentally

\[
H_{\text{CM}} / H_{\text{CM}} = 0.232. \quad \text{For the surface energy } \sigma_{\text{ns}} = H_{\text{CM}}^2 \Delta / 8 \pi \text{ we have for } \kappa = 0.158
\]

\[
\Delta = 6.5 \delta_\infty = 1.66 \cdot 10^{-8} V T_c / (T_c - T)
\]

\[
(\text{n is the concentration of "free electrons"}). \quad \text{If we use this expression, Eq. (1) takes the form}
\]

\[
x = 2.16 \cdot 10^7 \left( \frac{dH_{\text{CM}}}{dT} c T_c^2 \delta_\infty \right)
\]

For tin \( \delta_\infty (0) = 3.5 \cdot 10^{-6} \), according to references 10 and 6, whence \( \kappa = 0.149 \). The value \( \kappa \) is between 0.15 and 0.16 for tin agrees thus with sufficient accuracy both with experiments and with the requirements of the macroscopic as well as of the microscopic theory.

A further check must, in particular, consist in the measurement of a third effect: the change of \( \delta \) with field.2,7

The increase of \( \delta \) in tin near \( T_c \)
for an external constant field equal to \( H_0 = H_{CM} \)
must be \( 3\pi/4\sqrt{2} = 8.5\% \) (when we measure in a
weak variable field parallel to \( H_0 \)) and \( \kappa/4\sqrt{2} = 2.8\% \)
(when we measure in a weak field perpendicular to \( H_0 \)). We note also that a sharp change
in the behavior of Sn + In alloys takes place for
2.3\% In (see references 11 and 12) when \( \kappa \approx 0.6 \)
since, according to Chambers,\textsuperscript{13} the penetration
depth approximately doubles then. Theoretically,
however, the change in the properties must occur\textsuperscript{5,14} for
\( \kappa = 1/\sqrt{2} = 0.707 \). The well known
vagueness particular to experiments on alloys
makes it hardly possible to speak here about a
discrepancy between theory and experiments. In
any case, the agreement is appreciably better for
\( e_{eff} = 2e \) than for \( e_{eff} = e \).

If for Al we use in (1) the experimental value
\( \delta = \delta_{090}/\sqrt{T_C/\Delta T} \) with \( \delta_{090} = 4.93 \times 10^{-8} \) (and also
\( T_C = 1.17, |dH_{CM}/dT|_0 = 164 \) ) we get \( \kappa = 0.05 \).
In that case, however, \( \delta \neq \delta_L \) unless we deal with
values \( \Delta T \ll 10^{-3} \). We must therefore proceed
by two other ways. First we can determine \( \kappa \)
through the equation\textsuperscript{14} \( H_{CM}/H_{CM} = \sqrt{2} \) \( \kappa \) from
the experimental value\textsuperscript{8} \( H_{CM}/H_{CM} = 0.0363 \). Hence
\( \kappa = 0.0256 \) and, according to reference 7 and Eq. (2)

\[ \Delta \approx 628\delta = 445\delta(0)/\sqrt{T_C/\Delta T} = 10.9 \times 10^{-8} \sqrt{T_C/\Delta T}, \]

since we have \( \delta L(0) = 2.48 \times 10^{-8} \) if we use (3).
Experimentally\textsuperscript{9} \( \Delta = 9.0 \times 10^{-8}\sqrt{T_C/\Delta T} \) for
aluminum, i.e., we get excellent agreement with
theory.* Second, we should determine \( \delta_L(0) \) by
a less consistent method of comparison from inde-
pendent data; we make then additional assumptions
and, according to references 10 and 3, we have for
aluminum \( \delta L(0) = 1.6 \times 10^{-8} \) and from Eq. (3)
\( \kappa = 1.06 \times 10^{-3} \). Hence \( H_{CM}/H_{CM} = 0.015 \) and
\( \Delta = 18 \times 10^{-8}\sqrt{T_C/\Delta T} \) which disagrees with
experiments by approximately a factor two. Since
for aluminum and for other "Pippard" supercon-
ductors a comparison of the theory of reference 2
with experiments is difficult it is desirable for
this purpose to use in the first instance "London"
superconductors (at least for \( \Delta T \ll 0.1^\circ \)) and first of all lead.

We note in conclusion that it leads to difficulties
to use data on specimens of small dimensions
(films and so on) for a direct comparison of the
theory with experiments. This is connected with
the polycrystalline structure of such specimens
which leads to the impossibility to consider them
to be equivalent, thin specimens of the same metal
in bulk, but in single crystal form. One should
therefore in accordance with the well known con-
siderations of Pippard and others consider thin
polycrystalline films rather like alloys.

*We must note that the region of applicability of the theory
of reference 2 is considerably wider\textsuperscript{4} if we evaluate the values
of \( \Delta \) and \( H_{CM} \) than if we evaluate \( \delta \). We shall therefore for Al
use the necessary formul\ae of the theory of reference 2 also
for \( \Delta T \sim 0.1^\circ \) when

\[ \delta L/\kappa \sim 6 \times 10^{-8}/0.0256 = 2.3 \times 10^{-8} > \xi_0 \sim 10^{-9} \] (see references
5 and 10).

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Translated by D. ter Haar