ON SUPRACONDUCTIVITY I

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Summary

In this paper we give a phenomenological treatment of supraconductivity based on the fundamental laws of thermodynamics and on the assumption that the magnetic induction inside a supraconductive body is zero. After a brief survey of new experiments on supraconductivity (§ 1) these ideas are applied to a body, which is entirely in the supraconductive state (§ 2). In § 3 we consider the transition process for a needle parallel to a magnetic field and derive an equation connecting the jump of the specific heat and the derivative of the magnetic threshold value with respect to the temperature (Rutgers' equation). In § 4 these considerations are extended to a more general system of supraconductive bodies; in this case there will be a continuous transition process. In § 5 the disturbance of supraconductivity of a cylinder by a current flowing in the cylinder is considered in detail and Silsbee's hypothesis is confirmed. In § 6 we discuss our results and the experimental evidence for our fundamental assumptions. Some questions, which remain as yet unsolved, are mentioned.

§ 1. In the last year four different investigations, which gave rather unexpected results, have considerably increased our knowledge of phenomena, connected with supraconductivity.

Firstly Keesom and v. d. Ende [1]) observed a discontinuous change in the specific heat of tin at the transition temperature. This result has been confirmed by Keesom and Kok, who carried out a careful determination of the magnitude of the effect for tin and lately also for thallium [2]).

1) W. H. Keesom and J. v. d. Ende, Comm. Leiden, 219b.

2) W. H. Keesom and J. A. Kok, Comm. Leiden, 221e and 230c (Physica, 1, 175, 1934).

Secondly de Haas and his collaborators [1]) found the influence of a transversal magnetic field on the resistance of a wire of elliptic cross section, to depend on the orientation of the cross section in the field. The transition curves of the resistance for different orientations were different not only in the case, that the temperature was kept constant, but also in the case, that the temperature was varied, the external field remaining constant.

Thirdly Meissner and Ochsenfeld [2]) discovered, that the distribution of the magnetic field in the neighbourhood of a body changes rapidly, when the body becomes supraconductive in an external magnetic field; it seemed, that in one case this phenomenon could be described by assuming the supraconductor to have a magnetic susceptibility — $1/4\,\pi d$ (d being the density), but in another case (hollow leaden tube) the field appeared not to diminish inside the tube.

Fourthly de Haas and Mrs. Casimir [3]) studied the local distribution of the magnetic field inside a monocrystalline cylinder of tin at various external fields and temperatures. Prof. de Haas kindly allowed us to acquaint ourselves before the publication with the results, which show, that the magnetic field has the tendency to disappear entirely, when the external field or the temperature are low enough. The remaining internal magnetic field is inhomogeneous and disappears in a transversal field at first in the exterior parts of the cylinder.

All these new results, and some other results of apparently minor importance, require to be discussed thoroughly in order to see, in how far they allow new conclusions concerning the nature of the supraconductive state, and in how far they have to be considered as merely secondary effects.

The first result seems to make a thermodynamical treatment of the transition to the supraconductive state feasible. Following a suggestion of Langevin [4]), the transition can be described as a

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1) W. J. de Haas, Leipz. Vortr., 1933, 59; W. J. de Haas, J. Voogd and Miss J. Jonker, Comm. Leiden 229c (Physica, 1, 281, 1934).

The importance of such measurements has been emphasized by von Laue, see M. von Laue, Phys. Zs., 33, 793, 1932.

2) W. Meissner und R. Ochsenfeld, Naturw., 21, 787, 1933.

3) W. J. de Haas, and J. Casimir, Comm. Leiden 229d, Physica, 1, 000, 1934.

4) P. Langevin, Rapp. et Disc. du 1er Conseil Solvay, 311, 1911.

transition between two phases: the normal phase and the supraconductive phase.

A first trial in this direction has been made by Keesom [1]), long before the jump in the specific heat was discovered. After its discovery, Rutgers [2]), starting from Ehrenfest's consideration of phase transitions of second order, derived a relation between the jump in the specific heat and the derivative of the magnetic threshold value with respect to the temperature. This relation has been confirmed beautifully for tin and lately also for thallium, if the „longitudinal" threshold values are used.

One of us considered the state of affairs more in detail [3]) and arrived at the conclusion, that Ehrenfest's picture is not quite applicable to the transition to the supraconductive state. Limiting himself to those supraconductive states, where the induction B equals 0, he showed, that Rutgers' equation is identical with the statement, that the second law of thermodynamics applies to the magnetic disturbance, in spite of the fact, that the dying out of the so called persisting currents seems at first sight to be an irreversible phenomenon. He also found, that in non-longitudinal cases one has to expect, that, at external fields smaller than the longitudinal threshold value, some parts of the body may be in the normal state, while other parts are supraconductive.

In order to account for Meissner's and for de Haas' newer results, one of us recently suggested [4]), that in the supraconductive state B may always equal zero, this assumption clearing also unsolved questions in the previous thermodynamical treatment.

In the present paper these considerations will be worked out and extended in various respects. §§ 2 and 3 are essentially identical with considerations in A. § 4 extends the arguments of § 3, which apply to the longitudinal case, to bodies of arbitrary shape. In § 5 the disturbance by a current is considered and § 6 contains a discussion of results and as yet unsolved problems.

§ 2. Let us consider a metal body (weight 1 gram) in a homogeneous external field H_1 (a field, that would be homogeneous and

1) W. H. Keesom, Rapp. et Disc. de 4ième Conseil Solvay, 289, 1924.
2) P. Ehrenfest, Leiden Comm., Supp. 75b, see Nachtrag.
3) C. J. Gorter, Arch. Teyler, 7, 378, 1933 (quoted as A).
4) C. J. Gorter, Nature, 132, 931, 1933.

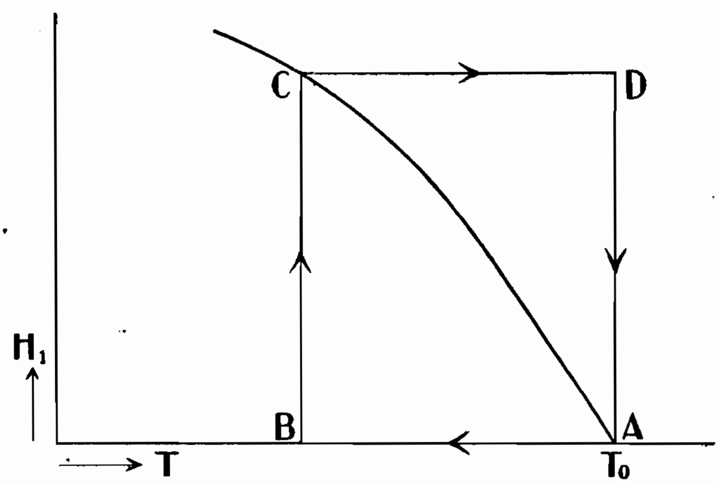
have the value H_1, if the body were not present, everything else remaining unchanged). In any point of the body, we can describe the state of things by the variables T, H_1 and B. H_1 and B are connected with the well-known variables H and P by the relations:

$$H = B - 4\pi P, \tag{1}$$

$$P = \chi Hd, \tag{2}$$

$$H = H_1 - \varepsilon P, \tag{3}$$

where d is the density, χ the magnetic susceptibility and ε the demagnetisation factor; this factor is constant throughout the body



if we have to do with an ellipsoid, and if χ is a constant. It is usual to introduce H and B as variables in the discussion of magnetic phenomena, but for several reasons we prefer in our case H_1 and B. One of these reasons is, that the work, done by the external field on the body has then a simple form; a second is, that it is often impossible to distinguish in a supraconductor between polarisation P and the effect of currents on the surface.

If we cool the body to a temperature below the normal transition-point in a zero external field and then apply a field H_1, which is not strong enough to disturb supraconductivity in any part of the body, we must expect such persisting currents to be induced on the surface of the body, that B remains zero at the inside of it. The same phenomenon can formally be described by assuming the magnetic susceptibility to be: $\chi = -1/4\,\pi d$. If an isolated body has not the shape of a ring, we can completely describe its behaviour in a field by putting: $\chi = -1/4\,\pi d$, or $B = 0$ inside the body. If, however, we have to do with a supraconductive ring, we must add the condition, that the total magnetic flux through the ring must remain zero.

It has been suggested previously (see § 1), that, if we start with a body in a magnetic field and then lower the temperature, the condition $B = 0$ will also be fulfilled in those parts of the body, which are in the supraconductive state. It is clear, that, if we consider the possibility, that supraconductive *rings* have been formed, the total flux through such rings will have to remain constant, but will not necessarily be zero.

The work, done by the external field on the body is $H_1\, d\sigma$ where σ denotes the total magnetic moment in the direction of H_1 [1]). Then the first law of thermodynamics yields:

$$dQ = dE - H_1 d\sigma, \tag{4}$$

where dQ is the supply of heat and dE the change of the energy of the body in the field. Introducing H_1 and T as independent variables, we get:

$$dQ = \left[\frac{\partial E}{\partial T} - H_1 \frac{\partial \sigma}{\partial T}\right] dT + \left[\frac{\partial E}{\partial H_1} - H_1 \frac{\partial \sigma}{\partial H_1}\right] dH_1. \tag{5}$$

If the body is and remains in the normal or in the supraconductive state, we can certainly apply the second law of thermodynamics (dQ/T) is a total differential), which yields:

$$\frac{\partial \sigma}{\partial T} = \frac{1}{T}\left[\frac{\partial E}{\partial H_1} - H_1 \frac{\partial \sigma}{\partial H_1}\right], \tag{6}$$

and so:

$$dQ = \left[\frac{\partial E}{\partial T} - H_1 \frac{\partial \sigma}{\partial T}\right] dT + T \frac{\partial \sigma}{\partial T} dH_1. \tag{7}$$

1) This is the work, done by the current in the coil, which brings about H_1. The magnetic interactionenergy ($\Delta \iiint_{-\infty}^{+\infty} (H^2/8\pi) dv$), amounting to: $+ H_1\sigma$ is then considered as belonging to the system. If on the contrary, we define our system in such a way, that this interaction energy does not belong to it, the expression for the work is: $-\sigma dH_1$. See: C. J. Gorter, Arch. Teyler, 7, 183, 1932, and also F. Bloch, Hb. f. Radiol., VI, II, 378, 1933, whose derivation however seems not quite correct. The following arguments remain essentially the same, if the latter expression (which is quite usual for electric polarisation) is accepted. The energy is merely diminished by $H_1\sigma$, and the place of the thermodynamical potential is taken by the free energy and vice versa.

In the supraconductive state $\partial\sigma/\partial T = 0$, and if we neglect the susceptibility of the normal metal (which is of the order of 10^{-6}) this certainly also applies to the normal metal. So we get from (7)

$$dQ = \left(\frac{\partial E}{\partial T}\right) dT \tag{8}$$

and from (6)

$$\left(\frac{\partial^2 E}{\partial H_1\, \partial T}\right) = H_1 \left(\frac{\partial^2 \sigma}{\partial H_1\, \partial T}\right) = 0 \tag{9}$$

and hence

$$dQ = c(T) dT. \tag{10}$$

So we find, that a change in H_1 is not accompanied by any caloric effect, and that the specific heat is independent of H_1.

§ 3. Let us now fix our mind upon the special case of a body having the shape of a very long ellipsoid (needle), orientated parallel to H_1. We know from de Haas and Voogd's investigations [1]), that in such cases, at least if we have to do with a single crystal, sharp transitions [2]) between the normal and the supraconductive states occur (longitudinal case), in contrast with the rather extensive intervals of transition, which are observed, if the field is perpendicular to the axis of a long wire.

In fig. 1 the transition curve has been drawn in the $H_1 - T$-diagram. Let us consider the cyclical process $ABCDA$ of this figure. From Keesom and Kok's measurements we know, that at $A(T = T_0)$ no heat of transition exists, but that the specific heats in the normal state c_n and in the supraconductive state c_s, are different.

Applying the first law of thermodynamics, (4) and (10) (except for the transitionpoint C, where the latter equation is not valid) we get:

$$-\int_{T_2}^{T_0} (c_s - c_n) dT + Q_2 = -\int_0^{\sigma_2} H_1\, d\sigma + H_2\, \sigma_2, \tag{11}$$

where the index 2 indicates the transitionpoint at C and Q_2 is the heat of transition at C.

From potentialtheory we know, that, if we admit $B = 0$ inside the

1) W. J. de Haas and J. Voogd, Comm. Leiden, 212c, 214c.

2) It is of course not certain, that, when the resistance has vanished, the whole body is in the supraconductive state, but for the limiting case of an extremely oblong ellipsoid this seems very plausible.

body, $\sigma = - H_1/(4\pi - \varepsilon)d$, where ε is the demagnetisationfactor. In the case considered $\varepsilon = 0$ and thus:

$$\sigma = - \frac{H_1}{4\pi d}, \tag{12}$$

and (11) becomes:

$$Q_2 = \int_{T_2}^{T_0} (c_s - c_n)\, dT - \frac{H_2^2}{8\pi d} \tag{13}$$

Up to now, we have hardly made any special hypothesis, but now we introduce the assumption, that the second law of thermodynamics applies also to the transition at C. Applying the second law to our cyclical process ($\int dQ/T = 0$), we get:

$$\frac{H_2^2}{8\pi d} = \int_{T_2}^{T_0} (c_s - c_n)\, dT - T_2 \int_{T_2}^{T_0} \frac{c_s - c_n}{T}\, dT. \tag{14}$$

The important equation (14) may also be obtained in a different way. In analogy to the usual transitions of phase, it may be useful to introduce the thermodynamical potential (Z-function; free energy at constant pressure, in our case at constant external field.) For the supraconductive state the Z-function will be:

$$Z_s = T \int_T^{T_0} \frac{c_s}{T}\, dT - \int_T^{T_0} c_s\, dT + \frac{H_1^2}{8\pi d} + AT + B, \tag{15}$$

where A and B are arbitrary constants; and for the normal state:

$$Z_n = T \int_T^{T_0} \frac{c_n}{T}\, dT - \int_T^{T_0} c_n\, dT + AT + B \tag{16}$$

If the body is in the supraconductive state $Z_s < Z_n$. If we increase the external field, Z_s increases and the transition to the normal state can occur when $Z_s = Z_n$; and it *will* occur at this value, if the transition is reversible. By equalising Z_s and Z_n we get equation (14).

Thus it seems possible to predict the magnetic threshold values H_2, if the difference of the specific heats in the normal and in the supraconductive state is known as a function of the temperature. For the immediate neighbourhood of the normal transitionpoint ($H_1 = 0$) we get:

$$\left(\frac{dH_2}{dT_2}\right)^2 = \frac{4\pi d\,(c_s - c_n)}{T_0}, \tag{17}$$

This is identical with Rutgers' equation. This equation has been verified for tin and for thallium [1]) and gives excellent agreement with the measurements. So it appears, that it was legitimate to apply the second law of thermodynamics to the transitionprocess at C. This result will be discussed in § 6.

§ 4. Let us now consider an arbitrary body in an external magnetic field H_1. As long as the body is completely in the supraconductive state, the Z-function [2]) is given by $Z_{s,0} + (1/8\pi d)\varkappa H_1^2$, but the constant $\varkappa$ is now different from unity, and depends in a complicated way on the shape of the body and its orientation in the field [3]). If there would exist a sharp transition of the body as a whole from the supraconductive to the normal state, one would expect this transition to occur when $Z_n - Z_{s,0} = (1/8\pi d)\varkappa H_1^2$ that is [4]) when $H_1 = \sqrt{\varkappa} H_2$.

Such a sharp transition, however, would be in disagreement with de Haas and Voogd's measurements on the resistance of a monocrystalline tin wire in a transversal magnetic field [5]). Also from a theoretical point of view one can hardly expect the transition to take place in this way, since it is quite possible, that part of the body may be in the normal state, another part remaining supraconductive. It seems reasonable to assume, that, as soon as the transition of a very small part of the body from the supraconductive state to the normal state will be accompanied by a decrease of the total Z-function, a continuous transition process will start.

We will now calculate the change of magnetic energy and the corresponding change of Z, when such an infinitesimal variation takes place.

The magnetic energy E_m, will be given by $-(1/8\pi d)\varkappa H_1^2$, for $E_m = \int H_1\, d\sigma$. On the other hand we have:

$$E_m = \frac{1}{8\pi} \iiint H^2\, dv_e - \frac{1}{8\pi} \iiint H_1^2\, dv_{e+i} \qquad (18)$$

The first integral is taken over the exterior volume, the second one also over the interior of the body. H denotes the field when the body is

1) W. H. Keesom and J. A. Kok, Physica, 1, 175, 1934, Leiden Comm. 230c.

2) $Z_{s,0} = T\int_0^T \frac{c_s}{T}\, dT - \int_0^T c_s\, dT$; $Z_n = T\int_0^T \frac{c_n}{T}\, dT - \int_0^T c_n\, dT$.

3) $\varkappa > 1$, e.g. for a sphere $\varkappa = 3/2$, for a cylinder in a transversal field $\varkappa = 2$.

4) $H_s^2/8\pi d = Z_n - Z_{s,0}$.

5) W. J. de Haas, and J. Voogd, Comm. Leiden 212c.

present. It is determined by the condition, that in any point at the surface of the body, the normal component of H is zero and that $H \to H_1$ at large distances from the body. We will assume that H_1, though practically homogeneous in a large region of space, is produced by a finite coil and therefore tends to zero in infinity more rapidly than r^{-2}. We will calculate the change of E_m, δE_m, when at a point on the surface of the body an element of volume δv is supposed to be transformed into the normal state. Such a change of shape of the supraconductive region will give rise to an additional $\delta H = \nabla \delta \Phi$ but to the first order of magnitude this will give no contribution to E_m, for

$$2 \iiint H \, \delta H \, dv_e = 2 \iiint (\nabla \, . \, H \, \delta \Phi) \, dv_e = 0. \tag{19}$$

because of the conditions imposed on H. On the other hand the element of volume δv is now an exterior element and gives a contribution to E_m:

$$\delta E_m = \frac{1}{8\pi} H_t^2 \, \delta v \tag{20}$$

where H_t is the value of the field H at the point considered. Since $E_m + \delta E_m$ is still of the form $(1/8\pi)\,(\varkappa + \delta\varkappa) H_1^2$, the corresponding change of Z will be given by $-(1/8\pi)\,(\varkappa + \delta\varkappa) H_1^2$ and thus by:

$$\delta Z = - \frac{1}{8\pi} H_t^2 \, \delta v \tag{21}$$

There is no difficulty in extending this theorem to a more general system of supraconductive bodies, including rings in an arbitrary magnetic field. It is convenient to regard the magnetic field as being produced by supraconductive rings [1]), that form part of our system. If we then calculate the change of total magnetic energy, when a variation in the shape of one of the bodies occurs taking into account the condition, that the magnetic flux through any ring must remain constant, we will find exactly the same result as when we introduce a Z-function and calculate δZ [2]).

Be i_k the current in the k^{th} ring, the magnetic energy will be of the form:

$$W_m = \tfrac{1}{2} \sum_{k,l} L_{kl} \, i_k \, i_l, \tag{22}$$

1) The idea of introducing supraconductive rings in order to avoid difficulties connected with the definition of magnetic energy is due to H. A. Lorentz.

2) For if the „external" work is always zero, the change in the Z-function has to be replaced by the change in the free magnetic energy.

where L_{kl} is an intricate function of the shape and orientation of all the supraconductors concerned. Put $p_l = (\partial W/\partial i_l)$, then:

$$p_l = \Sigma L_{kl} i_k = -\frac{1}{c} N_l, \tag{23}$$

where N_l is the flux through the l^{th} ring. It is now easily shown, (and a well known theorem in theoretical dynamics) that:

$$(\delta W_m)_p = -(\delta W_m)_i \tag{24}$$

$(\delta W_m)_i = \delta E_m$ may be calculated in exactly the same way as before, so we find again:

$$\delta Z = (\delta W_m)_p = -\frac{1}{8\pi} H_t^2 \, \delta v \tag{25}$$

Let us suppose, that in the element of volume δv supraconductivity has been replaced by the normal state. The total change of Z will be given by:

$$\delta Z = \delta v \left[(Z_m - Z_{s,0}) d - \frac{1}{8\pi} H_t^2\right] \tag{26}$$

This will be negative if $|H_t| > H_2$, the threshold value in the longitudinal case. As soon as this is the case the field will begin to penetrate into the body [1]).

When the field is increasing, the volume of the supraconductive parts of the body will gradually diminish. The boundaries of these parts (as far as they do not coincide with the surface of the body), will have to be such, that everywhere $H_t = H_2$. The transition process will come to an end when $H_1 = H_2$, for clearly there can exist no region where B equals 0 such, that $H_t < H_1$ at all points of the boundary.

Qualitatively these results are in agreement with de Haas and Voogd's experiments, but our considerations do not yet give a complete picture of the transition process. In § 6 we will make some further remarks on that problem.

§ 5. From Kamerlingh Onnes, Tuyn, de Haas and

1) We thus have given a thermodynamical derivation of the condition formulated by M. von Laue, Phys. Zs. 33, 793, 1932.

V o o g d's measurements [1]) it is known, that, if the measuring current in a wire is increased, the transition temperature shifts towards lower values. The magnitude of the shift is in good agreement with S i l s b e e's rule, stating that supraconductivity will be disturbed as soon as the magnetic field produced by the current at the surface of the wire equals the normal threshold value in a homogeneous field at the temperature considered.

We will try to consider this disturbance by a current, flowing in the supraconductor itself from our thermodynamical point of view.

We will suppose again, that inside the wire B equals 0; this will certainly be true if we suppose that the metal has first been cooled below the transition temperature and that afterwards the current has been switched on. The current will then flow along the surface of the wire.

Let us suppose that our supraconductor forms part of a very large, entirely supraconductive circuit, and ask at which value of the current the arrangement ceases to be stable. From the preceding section we know, that this will be the case as soon as the tangential field $H_t > H_2$. Then the field will penetrate into the body, supraconductivity will be disturbed near the surface and the current will be forced to flow deeper inside the wire. As the total current will scarcely be affected by this retiral, the magnetic field at the surface of the remaining supraconductive parts will even be higher than it was previously at the surface of the wire itself. So the supraconductive region, at the surface of which the current flows, will have to retire again and again, without being able to reach a state of equilibrium. This process can only terminate in the disturbance of the current, though the exact mechanism of this disturbance cannot be found from the above line of reasoning, which involves that the total magnetic flux through the whole circuit has always to remain constant (see § 6). Still it is satisfactory, that we can conclude to the existence of a maximum for the current, which cannot be surpassed without losing the possibility of a stable supraconductive system. S i l s b e e's rule is defined more precisely by our result, since our condition is, that the magnetic field at the surface cannot be greater than the longitudinal threshold field (the difference between longi-

1) H. K a m e r l i n g h O n n e s, Comm. Leiden 133a.
W. T u y n and H. K a m e r l i n g h O n n e s, Comm. Leiden 174a.
W. J. d e H a a s and J. V o o g d, Comm. Leiden 214c.

tudinal and transversal disturbance was not known at the time the rule was formulated).

It is possible to arrive at the same result, without explicitely making use of the picture of an entirely supraconductive circuit. As this derivation is rather instructive in as far as it illustrates another aspect of the mechanism of the disturbance, we will also give it here.

In § 4 we have seen, that, if we define our system in such a way that no external work is done, the Z-function is replaced by the free energy, the transition of a small element of volume δv to the normal state occurring as soon as the corresponding change in the total magnetic energy is equal to or greater than the corresponding difference in the thermal free energy $d(Z_n - Z_{s,0})\,\delta v$. The magnetic energy here only comprises the energy of selfinduction and of mutual induction of the persisting currents.

In analogy to this formulation we can expect, returning to a cylindrical wire carrying a current, that supraconductivity will retire from a volume δv if the corresponding decrease of the energy of the current will be larger than the increase of the thermal free energy.

The „change of the energy of the current" can be derived from the well known expression for the force exerted by two parallel elements of current I_z and J_z on another. This force, acting perpendicular to the currents, is:

$$\left(I_z J_z \frac{\partial}{\partial x}\frac{1}{r},\; I_z J_z \frac{\partial}{\partial y}\frac{1}{r},\; 0\right)$$

where $r(x, y, z)$ indicates the line connecting the elements. As long as we keep z constant, this force may be derived formally from a „mutual potential energy" $W_m = - I_z J_z/r$. By integration of this expression we can calculate the total „mutual potential energy" of all the elements of a current flowing homogeneously on the surface of a cylinder of radius R and length L. We obtain, if $R \ll L$

$$W_m = - i^2 L \log 2 L/R \tag{27}$$

where i denotes the total current. The increase of this energy when R decreases by δR is:

$$\delta W_m = - \frac{Li^2}{R}\,\delta R = - \frac{i^2}{2\pi R^2}\,\delta v \tag{28}$$

We have seen, that it has to be expected, that supraconductivity will be disturbed if this $-\delta W_m > \delta F_t$. This leads to

$$\frac{L^2}{2\pi R^2 d} > \int (c_s - c_n)\, dT - T \int \frac{c_s - c_n}{T}\, dT.$$

We have found in § 3 that

$$H_2^2/8\pi d = \int_T^{T_0} (c_s - c_n)\, dT - T \int_T^{T_0} \frac{c_s - c_n}{T}\, dT$$

and further we know, that

$$H_t = 2\, i/R,$$

so our condition becomes:

$$H_t^2 > H_2^2$$

in agreement with the result deduced by our first method.

§ 6. In the preceding paragraphs we have tried to give a consistent thermodynamical treatment of the transitions from the supraconductive state to the normal state, supposing B to equal zero in the supraconductive state. From the validity of Rutgers' equation we concluded, that apparently the second law of thermodynamics applies to the transition. This result suggests, that the transition is essentially reversible, which would mean, that, whenever a part of a body becomes supraconductive, such persisting currents are started, that the external field will be screened off, in order that $B = 0$ inside the supraconductive part.

This hypothesis was proposed by one of us after the appearance of Meissner and Ochsenfeld's publication on the distribution of the magnetic field in the neighbourhood of a supraconductor. However, already de Haas, Voogd and Miss Jonker's result on the transversal disturbance in tin wires of elliptic cross-section contained an indication in favour of this hypothesis. Von Laue's idea, that the beginning of the disturbance is determined by the maximal tangential value of the magnetic field, a supposition, which in § 4 could be justified by our thermodynamical treatment, explains why the beginning depends on the orientation of the transversal field with respect to the elliptic wire. For this explanation the existence of persisting currents is essential, and the result, that also in a constant field the transition curve of the resistance depends on the orientation, indicates, that also in this case we have to do with persisting currents (or, what amounts to the same: large negative susceptibilities).

It is well known, that Meissner's and Ochsenfeld's measurements of the field between two parallel tin cylinders in an external transversal field are in very good agreement with the assumption $B = 0$.

Also de Haas and Mrs. Casimir's recent measurements on the distribution of the field inside a tin cylinder prove, that the magnetic field undergoes important changes when the cylinder becomes supraconductive and that in some regions B vanishes completely.

Though certainly the assumption $B = 0$ cannot be considered as rigourously proved by all these measurements, this assumption offers undoubtedly the most simple and elegant way of explaining qualitatively the phenomena observed, with which it is never in contradiction.

An apparent difficulty is furnished by Meissner and Ochsenfeld's result, which is confirmed by de Haas and Mrs. Casimir's observation, that sometimes far below the transition-temperature and even in the absence of an external field, regions in the body may exist, where certainly $B \neq 0$. It seems to us, that this may be ascribed to supraconductive rings inside the body, as certainly the conditions in these experiments favoured the formation of such rings. Inside the rings the field due to the current along the rings may prevent the transition to the supraconductive state. Probably also Kamerlingh Onnes and Tuyn's observations on persisting currents in a sphere may be ascribed to such rings.

A second problem, which is not yet completely solved, is the mechanism of the gradual disturbance of supraconductivity (e.g. in a transversal magnetic field). One could imagine, that, as soon as the tangential magnetic field will surpass the critical value, supraconductivity will retire from the surface to deeper regions of the body. This picture, however, is not satisfactory, for then, in general, the field at the surface of the body would decrease, so that, at least if the transitions are really reversible, at the surface new supraconductive regions would be formed. So it seems better to imagine, that part of the supraconductor will be perforated or reduced to pieces, rather than to suppose that there will exist a sharp retiring boundary between the two phases. The mean magnetic field in larger regions will then have some value between zero and the maximum value, in agreement with de Haas and Mrs. Casimir's results. In practice the condition $B = 0$ may thus perhaps lose its rigour in the neighbourhood of the transition line.

We hope, however, to be able to treat the problems of gradual disturbance together with the phenomena of hysteresis and the formation of rings as well as some more purely theoretical questions in a second paper.

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