Levitation of a magnet over a flat type II superconductor

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Levitation of a magnet over a type II superconductor where the field at the superconductor exceeds H_{c1} is described and shown. The penetration and pinning of the flux lines in the superconductor cause the position of the magnet to be stable over a flat disk; a complete Meissner effect would make this position unstable. Furthermore, the observed dependence of the height of levitation on such variables as the thickness of the superconducting disk and the size of the magnet are consistent with a model described in this paper based on the energy cost of flux penetration through vortices and inconsistent with a Meissner effect model.

INTRODUCTION

It has long been known that it is possible to levitate a magnet over a superconductor or vice versa due to expulsion of the magnetic field from the interior of the superconductor.[1-3] These experiments were generally performed using a bowl-shaped superconductor over which the magnet floats or a magnetic disk, magnetized normal to the surface over which the superconductor floats. A current-carrying circular loop can replace the magnetic disk.[4] In the case of the bowl, the shape provides a gravitational minimum leading to lateral stability. The lateral stability in the case of the magnetic disk or current-carrying coil is the result of a minimum in the perpendicular magnetic field at the symmetry axis of the coil or disk. This minimum only applies when the height of the superconductor above the plane of the coil or disk is much smaller than the radius of the coil or disk.[5]

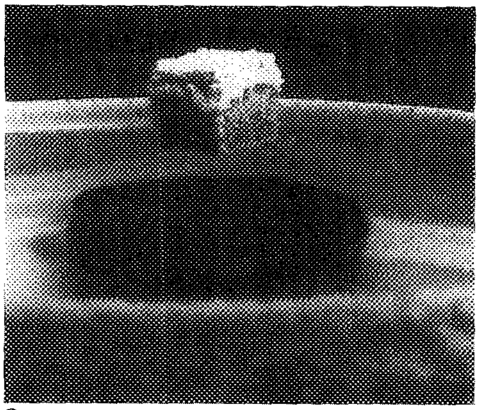
It has generally been considered necessary to use a type I superconductor or a type II superconductor below H_{c1} where a complete Meissner effect exists. We show in this paper, both experimentally and theoretically, that levitation works equally well with a type II superconductor between H_{c1} and H_{c2}. To our knowledge, this calculation has not previously been made. Furthermore, the penetration of the magnetic flux lines together with even a low vortex pinning force provides lateral stability so that a magnet can float stably over a flat disk.

The advent of the new high T_c superconductors has made possible experiments which were previously difficult. The fact that the transition temperature is well above 77 K allows measurements to be performed in a petri dish partially filled with LN_2. Direct, convenient measurements of the local fields around the superconductor can now be made with a Hall probe and with ferromagnetic powders.

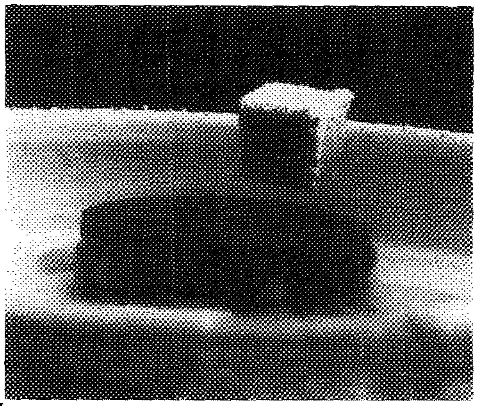
Levitation of a magnet over one of the new oxide superconductors was first publicly announced by Maple.[6] A LaSrCuO compound was used which still required cooling below LN_2 temperatures. The newer $Y_1Ba_2Cu_3O_{9-\delta}$ compounds have transition temperatures above 90 K[7] and are extreme type II, with a penetration depth λ estimated to be near 1400 Å and a Ginzburg–Landau coherence length near 22 Å at $T = 0$.[8]

Figure 1 shows the levitation of a Nd-Fe-B cubic magnet 0.7 cm on a side over a sintered, O_2-annealed, 2.5-cm-diam, 0.6-cm-thick disk of single phase $Y_1Ba_2Cu_3O_{9-\delta}$. The magnet is a permanent bar magnet; its moment is oriented parallel to the surface of the superconducting disk. The height h at which it floats (defined as the distance between the top surface of the superconductor and the bottom surface of the magnet) is dependent on the thermal and magnetic history; shown is the maximum height of 0.64 cm. It can be made laterally stable when positioned essentially anywhere over the disk; two examples are shown in Figs. 1(a) and 1(b).

The perpendicular field at the surface of the supercon-



a



b

FIG. 1. Levitation of a cubic Nd-Fe-B magnet 0.7 cm on a side over a flat 2.5-cm-diam, 0.6-cm-thick $Y_1Ba_2Cu_3O_{9-\delta}$ superconducting disk. Magnet initially positioned at (a) the center of the disk, (b) the edge of the disk.

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ductor, measured with a Hall probe, with the magnet levitated 0.64 cm above, was less than 50 G directly under the center of the magnet, 600 G directly under the edges of the magnet and 200 G at the edge of the disk. The field tangential to the superconductor maximizes directly under the center of the magnet and is equal to approximately 800 G. The tangential field inside the superconductor may be reduced by surface screening currents. The perpendicular field, however, must be continuous. H_{c1} of this material, measured at 77 K in a vibrating sample magnetometer for a long cylinder of identically prepared material with the field parallel to the long axis of the cylinder, is approximately 100 G.[9] This number is in agreement with the value given by Cava *et al.* in Ref. 8. Thus, the fields inside the superconductor exceed H_{c1} everywhere except perhaps at the center. The resulting vortices will have an areal density proportional to the field. Since the superconductor is far below H_{c2}, which is estimated to be near 300 kG at 77 K,[8] the density of vortices will be low.

There are two fundamental questions concerning the levitation. First, why does the magnet levitate and second, why is its position laterally stable. We will address the former question first. An accurate calculation including both partial flux expulsion and partial penetration is quite complex since the field from the permanent magnet is spatially nonuniform. Therefore, we will consider two limiting cases and determine the height predicted by each. The one limit is complete flux penetration, the other is complete flux expulsion. We will show that predictions of the flux penetration model are quite close to what is experimentally observed.

MODELS

The complete flux expulsion is modeled by considering a magnetic sphere with moment M above a superconducting plane. The boundary condition at the plane, given a Meissner effect, is that the perpendicular field $(H_\perp) = 0$. This condition may be met in the usual way by considering an image sphere identical to the real sphere and equidistant below the surface. The magnetic moment points parallel to that of the real sphere and parallel to the surface of the superconductor. The external magnetic field of a magnetized sphere, in polar coordinates referenced to its magnetic moment direction, is given by the gradient of $(-4\pi MR^3 \cos\theta/3r^2)$ where R is the radius of the sphere.[10] The energy of the real sphere in the magnetic field of the image sphere is just

$$E = -MHV = 2\pi^2 M^2 R^6/9d^3$$

if we approximate H as constant throughout the real sphere and equal to its value at the center of the sphere. Here, $d \equiv h + R$ is the distance from the center of the sphere to the top surface of the superconductor and V is the volume of the sphere. There is also the gravitational energy of the sphere, which is just $mgh = \rho Vg(d-R)$ where m is the mass of the sphere and ρ the density. Differentiating the total energy with respect to d to find the stable point, we obtain

$$d = (\pi M^2 R^3/2\rho g)^{1/4}.$$

This gives $h \simeq 1.0$ cm, where we obtained the value of M (400 G) directly by measuring the field at the surface of the magnetic cube. Note that d is independent of the thickness of the superconductor as long as the thickness is greater than the penetration depth λ.

Now we will consider flux penetration through vortices. We will calculate the perpendicular field due to the magnet at the superconductor assuming no distortion of field lines. This model is obviously an extreme limit; between H_{c1} and H_{c2} there is still partial flux expulsion. It is, however, empirically observed that the fields due to the magnet are not in fact significantly disturbed by the presence or absence of the superconductor. If vortices in the superconductor were strongly pinned, the critical state model of Bean[11] would be applicable, leading to a distortion of the field lines. For sufficiently strong pinning, the field would in fact be excluded from the interior of much of the superconductor. In this limit, the Meissner model described above is likely to provide a better description. This material, however, has a low J_c; at 77 K and 600 Oe, J_c is estimated to be less than 5 A/cm^2,[12] hence the pinning is low. Levitation of the magnet is due to the energy cost of putting vortices into the superconductor. We will assume that the structure of a vortex is as presented by Tinkham[13] for extreme type II behavior where $\xi \ll \lambda$.[14] We will neglect the interaction of the vortices since $H \ll H_{c2}$.

If we consider a spherical magnet with moment M oriented parallel to an infinite superconducting plane a distance $d \equiv h + R$ below, then the total integrated flux through each half plane (since the magnetic field will be oppositely directed for $x > 0$ and $x < 0$):

$$\begin{aligned} |\Phi| &= \int_{-\infty}^{\infty} dy \int_0^{\infty} dx\, B_z \\ &= 4\pi MR^3 d \int_{-\infty}^{\infty} dy \int_0^{\infty} \frac{x\,dx}{(x^2+y^2+d^2)^{5/2}} \\ &= 8\pi MR^3/3d\,. \end{aligned}$$

The x, y, z coordinate system has been defined such that the origin coincides with the center of the sphere, the x axis is along the moment and the z axis is perpendicular to the plane of the superconductor. The total flux through the plane must be divided between N vortices each containing a flux quantum Φ_0. Therefore,

$$N = 2|\Phi|/\Phi_0 = 16\pi R^3 M/3\Phi_0 d\,.$$

The energy of a vortex per unit length, ϵ_1, is given by Tinkham as[13]

$$\epsilon_1 = H_{c1}\Phi_0/4\pi\,.$$

So, the total energy of putting vortices through the superconductor

$$E = N\epsilon_1 L = 4R^3 M H_{c1} L/3d\,,$$

where L is the thickness of the superconductor. This expression assumes that the vortices pass straight through the superconductor, an approximation which is only strictly valid for $L \ll R$. Again this energy must be balanced against the gravitational energy of the magnet $mg(d-R)$. Differentiating with respect to d to get the height at which the magnet is stable and substituting $\frac{4}{3}\pi R^3\rho$ for m, we find

$$d = (MH_{c1}L/\pi\rho g)^{1/2},$$

which gives $h = 0.7$ cm.

EXPERIMENTAL RESULTS AND DISCUSSION

The measured height of 0.64 cm is remarkably close to the height predicted by the flux penetration model, 0.7 cm. Several approximations were made in the calculation which are not strictly valid in our experiment. The superconducting disk is not an infinite plane. The magnetic field does of course fall off rapidly with distance making this distinction less relevant. In fact, the height of levitation over a disk 5 cm in diameter is identical to the height over the 2.5-cm disk, indicating that this approximation is sufficiently valid. Secondly, the magnet is not spherical but cubic. However, the leading term in a multipole expansion will be the same. Finally, the thickness L is not much smaller than the magnet dimension R. Measurements, with the magnet floating 0.64 cm above the superconductor, of the magnetic flux entering at the top surface and exiting at the bottom surface indicate that some flux lines must close inside the superconductor. Thus, the approximation of vortices passing straight through is not completely satisfied. However, since L is comparable to R, the vortex length in closing through the superconductor is comparable to the length passing straight through, hence, the calculation is still approximately justified.

The height predicted by the flux penetration model is dependent on L, unlike the flux expulsion model. Levitation was measured over three thinner superconducting disks of thickness 0.015, 0.11, and 0.28 cm. The height h of the magnet over these was 0.0, 0.2, and 0.5 cm, respectively. Thus, for the four thicknesses, d does go approximately as $\sqrt{L}$. Note that the thinnest disk is still far thicker than the penetration depth at 77 K, and yet it did not cause the magnet to levitate at all.

Levitation was also measured over the four disks for three additional cubic magnets: 0.35, 0.6, and 1.2 cm on a side. As predicted by the flux penetration model, d is roughly independent of R. Since $h = d - R$, the height of the magnet above the disk actually *decreases* with increasing R. In fact, the largest magnet does not levitate at all over the three thinner disks; the separation d predicted by the flux penetration model is less than R. Note that it was necessary to use a 5-cm-diam 0.6-cm-thick disk with the 1.2-cm cubic magnet; the height then was 0.2 cm. When levitated over the 2.5-cm-diam 0.6-cm-thick disk, h was less, indicating that significant flux was closing outside the superconductor. The results presented in these two paragraphs are completely inconsistent with the predictions of the Meissner state model.

lations, hence the magnet would be expected to be unstable with respect to motion parallel to the flat surface of the superconductor. However, the vortices in the flux penetration model are, to some extent, pinned. Experimentally, it is observed that for lateral stability, the location of the trapped vortices must be established by bringing the magnet close to the superconductor before releasing it or by cooling the superconductor with the magnet resting on top of it. If the magnet is laterally unconstrained as it is brought towards the superconductor, as for example if it is resting on a glass slide, it will in fact slide off to one side. This instability occurs because the field at the superconductor is less then H_{c1}; the Meissner effect produces unstable levitation over a flat surface. By constraining the magnet laterally until H_{c1} is exceeded, vortices are introduced. Subsequent attempts to move the magnet require either the creation of new vortices or relocation of the old, which requires overcoming the pinning forces. This additional required energy produces the stabilization force. Small excursions parallel to the direction of the moment may be made about the stable position by allowing the moment of the magnet to follow the field lines. The magnet is then tilted with respect to the surface of the superconductor, as shown in Fig. 2. Excursions parallel to the plane of the superconductor but in a direction perpendicular to the moment require vortex creation or motion; the magnet does not tilt in this case.

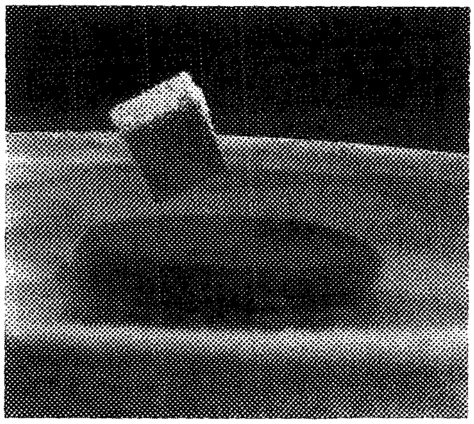

FIG. 2. Levitation of the magnet moved to one side of the disk after floating stably as in Fig. 1(a). The magnetic moment is tipped along the field lines of the trapped flux.

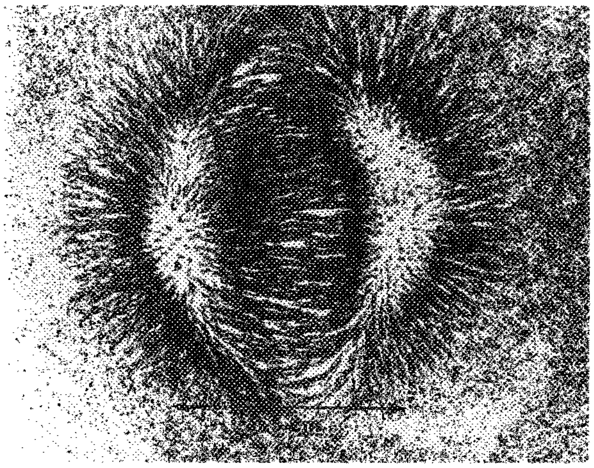

FIG. 3. Permalloy (Ni-Fe) powder pattern showing flux lines remaining in superconductor after the magnet is lifted away.

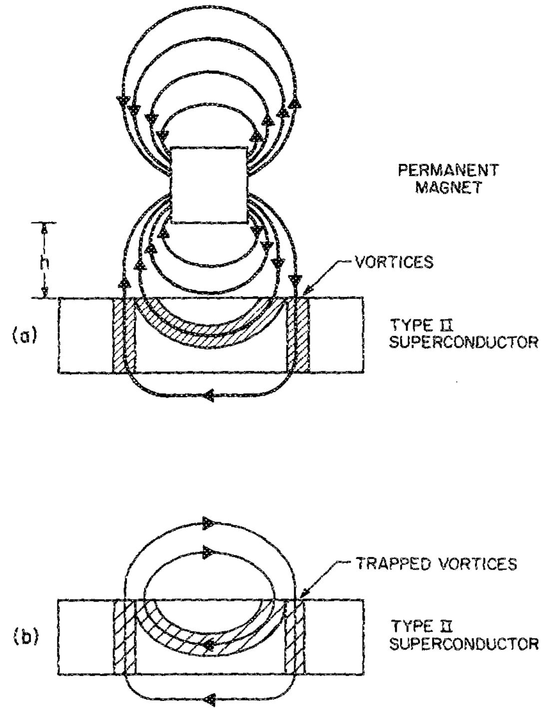

FIG. 4. Schematic of flux lines causing levitation and lateral stability of magnet over type II flat superconductor. (a) Magnet floating above superconductor. Some magnetic flux is expelled from the superconductor, some penetrates through and some flux lines close inside. (b) Magnet removed leaving the trapped flux which is 1–2 orders of magnitude less than the flux entering the superconductor with the magnet in place.

On removing the magnet, the remnant trapped vortices may be measured. The residual fields are 10–20 Oe with the maximum perpendicular fields approximately 1.5 cm apart. The field is parallel to the surface at the middle. Using Permalloy (Ni-Fe) powder, the field lines of these residual trapped vortices may be graphically demonstrated, as in Fig. 3.

Figure 4 summarizes the model for levitation and lateral stability. Figure 4(a) shows schematically the flux lines partially penetrating and partially excluded with the magnet in place. Figure 4(b) shows the trapped flux lines left when the magnet is taken away.

CONCLUSIONS

In conclusion, we have demonstrated both theoretically and experimentally that a complete Meissner effect is not necessary for levitation and, indeed, is inconsistent with experimental observations. A model for levitation based on the energy cost of vortices in the superconductor provides remarkably good predictions of both the height of levitation and the dependence of the height on the thickness of the superconductor and size of the magnet. Flux penetration makes possible the stable levitation of a magnet over a flat superconducting disk. If the pinning force for the vortices were increased due to improved material properties, the lateral restoring force would become larger, permitting potential application.

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